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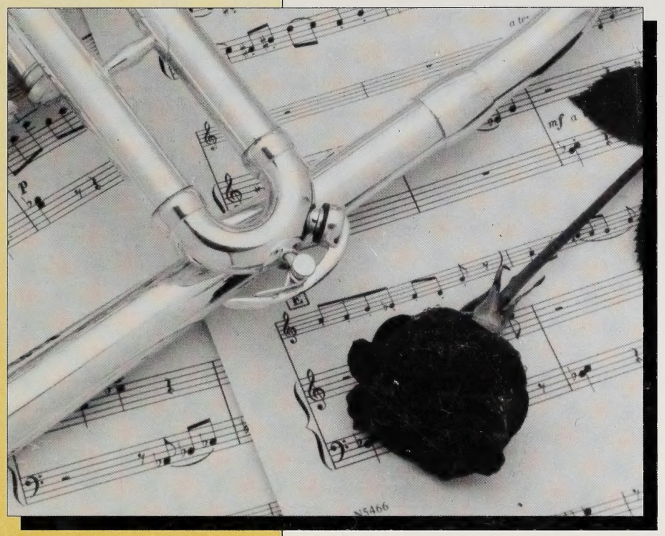

8

MODULE 3
FRACTIONS AND DECIMALS

Distance Learning

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Mathematics 8

Module 3: Fractions and Decimals

MODULE BOOKLET

Mathematics 8
Student Module
Module 3
Fractions and Decimals

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Welcome to Module 3!

We hope you'll enjoy your study of ***Fractions and Decimals****.*

To make your learning a bit easier, a teacher will help guide you through the materials.

So whenever you see this icon,

turn on your audiocassette and listen.

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MODULE INTRODUCTION

What Lies Ahead

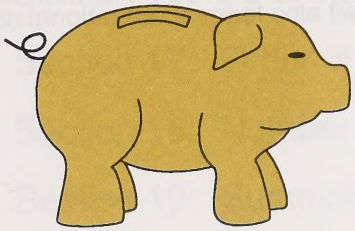
In this Module Introduction you will preview the module and learn how the module will be evaluated.

Working Together

In Module 2 you learned about whole numbers and integers. In this module you will learn about fractions and decimals. You will also learn about rational numbers.

Fractions and decimals have many different but equivalent forms. In some situations either the fraction or decimal number form is acceptable. In other situations the fraction form is more commonly used. In still other situations the decimal number form is preferred.

The decimal number form is used in our money system.



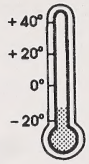
Usually numbers are rounded to the nearest hundredth of a dollar.

For example, a float costs $1.25.

Fractions are also used in our money system. $0.25 is sometimes referred to as a quarter.

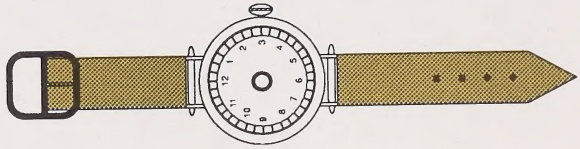
The decimal number form is used in the metric system of measurement. For example, a lobster weighs 1.5 kg.

The temperature is –13.5°C.

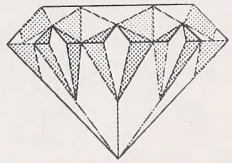


The fraction form is used in non-metric systems of measurement.

For example, it will take half an hour to complete a task.



The diamond is $1\frac{1}{2}$ carats in size.



Because the metric system is now becoming more common, there are fewer examples of the fraction form in everyday life. This, however, does not mean the fraction form is no longer important.

The fraction form is an important means of expressing division.

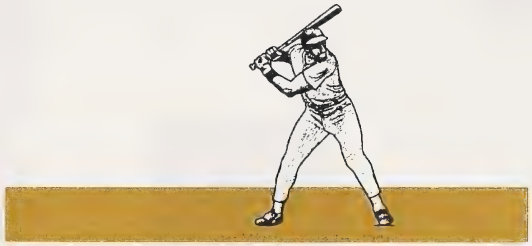
One pie is divided into six parts. Each of the six people received $\frac{1}{6}$ of the pie.

This is read as one-sixth.

The fraction form is also used in comparisons.

The child's age is $\frac{1}{9}$ of her mother's age.

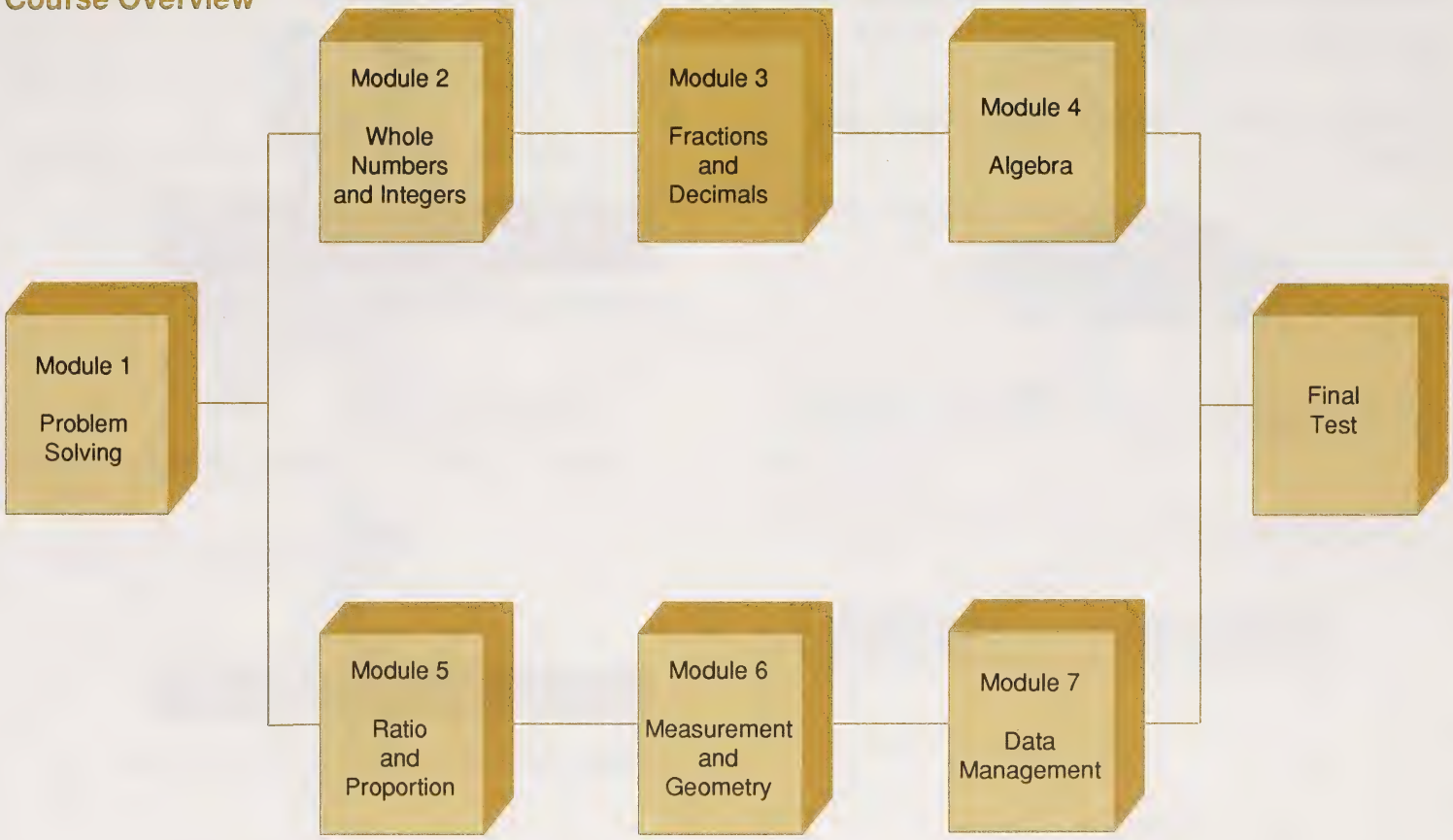
This is read as one-ninth.



The batter safely hit the ball $\frac{3}{5}$ of his times at bat.

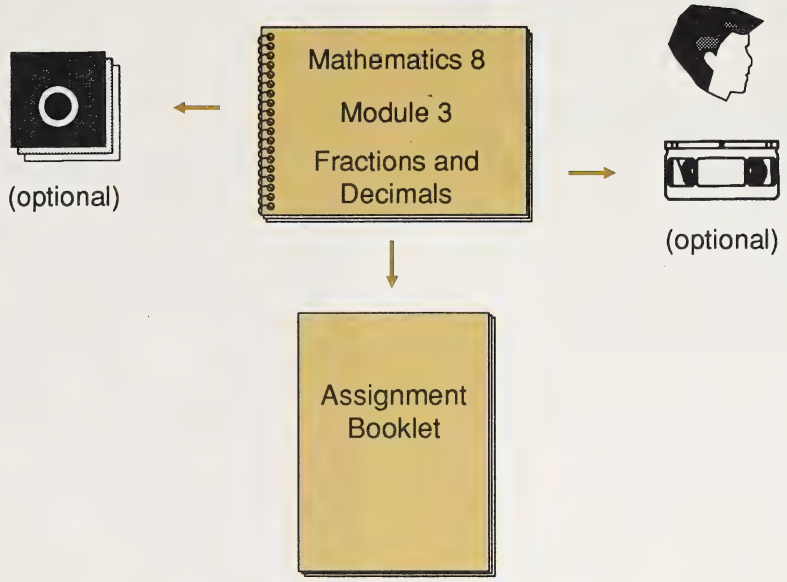
This is read as three-fifths.

Course Overview



Mathematics 8 has seven modules and a final supervised test. This module booklet is part of Module 3.

Module 3 Components



This module booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module. The computer and video activities in this booklet are optional; there are print alternatives. You should see your learning facilitator to check your answers to the activities in this booklet.

This module booklet is not to be submitted for a grade. **Your mark on this module will be determined by your work in the assignment booklet.**

Take time to preview the module booklet before beginning Section 1.

What Lies Ahead

In this section you will review these skills.

- recognizing place value of decimal numbers
- rounding decimal numbers
- comparing and ordering decimal numbers
- performing operations on decimal numbers
- applying the rules for order of operations

Working Together

You now have been working with decimals for several years. This section will review your skills with decimal numbers.

Review

1. Circle the diagram to the right that represents 0.102 modelled with base 10 blocks.

represents 1

represents 0.1

represents 0.01

represents 0.001

2. Round the following decimal numbers to the nearest hundredth.

   a. 0.325

   b. 0.674

   c. 0.439

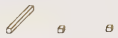
3. Arrange these decimal numbers in **ascending order** from least to greatest.

   0.25, 0.175, 0.251, 0.24, 0.3, 0.18

Space for Your Work

1. a.

b.

c. 

d.

4. Model each of the following using base 10 blocks.

Space for Your Work

a. $0.36 + 0.75$

b. $0.82 - 0.74$

c. 0.36×2

d. $1.36 \div 4$

5. Estimate each of the following. Then calculate the exact answers. Do not use calculators.

Space for Your Work

a. $8.3664 + 0.32 + 0.008$

b. $6 - 3.029$

c. 7.39×1.8

d. $33.4 \div 25$

e. $2.94 \div 0.15$

6. Explain how you would use a calculator to find the remainder for $1394 \div 25$.

7. Simplify each of the following.

Space for Your Work

a. $2.4 \div (1.6 - 0.4) + 0.3 \times 0.7$

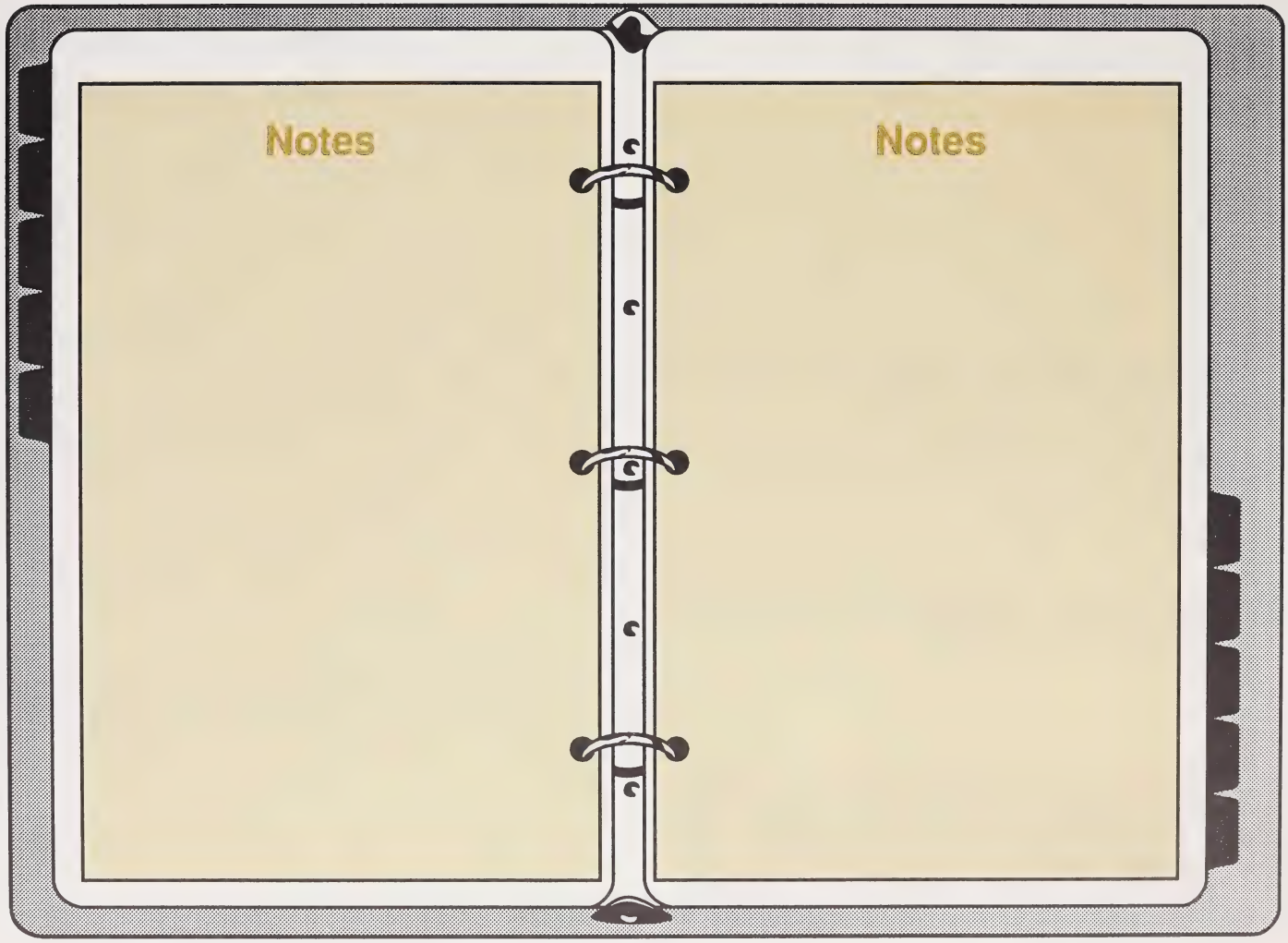
b. $3.2 + 0.8 \div 0.4$

8. Write a number sentence to describe each of the following and then simplify each one.

a. Add 0.12 and 0.7 and divide the sum by 0.02.

b. Divide the sum of 0.6 and 0.25 by the difference of these same two numbers.

See your learning facilitator to check your answers and to receive further instructions.


Notes
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What Lies Ahead

This section will test these skills.

- interpreting a fraction
- writing equivalent fractions
- writing basic fractions
- writing mixed numbers as improper fractions
- writing improper fractions as mixed numbers
- expressing a fraction as a decimal number
- expressing terminating decimals as fractions
- comparing fractions
- estimating sums, differences, products, and quotients
- computing sums, differences, products, and quotients using paper and pencil methods and using a calculator
- computing sums, differences, products, and quotients mentally
- performing a series of operations

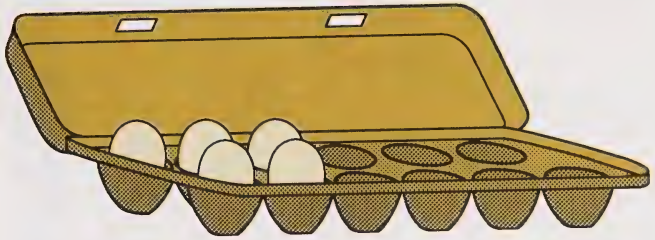
Working Together

The pretest in this section will help you and your learning facilitator determine your strengths and weaknesses with fractions.

Pretest

Space for Your Work

1. What fraction of this carton of eggs is full?



2. a. Write an equivalent fraction for $\frac{7}{9}$.

   b. Find the basic fraction for $\frac{14}{21}$.

   c. Express $\frac{75}{13}$ as a mixed number.

   d. Express $2\frac{1}{8}$ as an improper fraction.

   e. Express $\frac{4}{125}$ as a terminating decimal.

   f. Express $\frac{5}{9}$ as a repeating decimal.

   g. Express 0.135 as a fraction.

   h. Will $\frac{9}{14}$ produce a terminating decimal or a repeating decimal?

3. Arrange these fractions in **descending order** from greatest to least.

$\frac{5}{8}, \frac{7}{10}, \frac{3}{4}, \frac{1}{2}, \frac{11}{20}$

Space for Your Work

4. Estimate the following sums or differences and then calculate the exact answers. Do not use a calculator. Express each answer in simplest form.

   a. $\frac{5}{8} + \frac{2}{8}$

   b. $\frac{3}{6} - \frac{1}{6}$

   c. $1\frac{1}{10} + 2\frac{3}{10}$

   d. $2\frac{3}{5} - 1\frac{4}{5}$

5. Estimate the following sums and differences and then calculate the exact answers. Do not use a calculator. Express the answers in simplest form.

Space for Your Work

a. $\frac{3}{5} + \frac{1}{4}$

b. $\frac{7}{8} - \frac{3}{4}$

c. $1\frac{1}{6} + 2\frac{4}{9}$

d. $3\frac{3}{10} - 1\frac{2}{5}$

6. Compute the following operations mentally. Express the answers in simplest form.

a. $4\frac{3}{8} + 2\frac{7}{8}$

b. $2\frac{1}{4} - 1\frac{3}{4}$

c. $1\frac{1}{3} + 5\frac{7}{9}$

d. $3\frac{1}{5} - 1\frac{9}{10}$

7. Estimate the following products and then calculate the exact answers. Do not use a calculator. Express the answers in simplest form.

Space for Your Work

   a. $\frac{1}{5} \times \frac{1}{6}$

   b. $\frac{3}{7} \times \frac{5}{8}$

   c. $1\frac{3}{4} \times 2\frac{1}{3}$

   d. $1\frac{2}{3} \times 1\frac{3}{4}$

8. Multiply each of the following. Use cancelling where possible. Express the answers in simplest form.

   a. $\frac{7}{8} \times \frac{2}{5}$

   b. $\frac{3}{4} \times \frac{8}{9}$

   c. $1\frac{1}{2} \times 1\frac{5}{6}$

   d. $2\frac{7}{9} \times 3\frac{3}{5}$

9. Multiply mentally to obtain the exact answers. Express the answers in simplest form.

   a. $4 \times 3\frac{1}{2}$

   b. $\frac{1}{2} \times 2\frac{4}{5}$

   c. $3\frac{1}{2} \times 2\frac{2}{3}$

10. Estimate the following quotients and then calculate the exact answers. Do not use calculators. Express the answers in simplest form.

Space for Your Work

a. $\frac{3}{5} \div \frac{1}{5}$

b. $\frac{2}{3} \div 2$

c. $\frac{3}{4} \div \frac{3}{8}$

d. $\frac{4}{5} \div \frac{1}{2}$

e. $1\frac{3}{4} \div \frac{1}{4}$

f. $2\frac{1}{3} \div 1\frac{1}{2}$

11. Write the reciprocals of each of the following fractions.

a. $\frac{2}{5}$

b. 3

c. $1\frac{1}{3}$

12. Two numbers are reciprocals of each other. One number is 16 times larger than the other. Find the two numbers.

13. Mentally divide each of the following. Express each answer in simplest form.

    a. $6\frac{3}{8} \div 3$

    b. $8\frac{4}{5} \div 3$

    c. $5\frac{1}{3} \div 5$

14. Find the following sums. Use shortcuts. Express each answer in simplest form.

    a. $\frac{3}{7} + \frac{1}{4} + \frac{2}{7} + \frac{3}{4}$

    b. $\frac{1}{3} + \frac{2}{5} + \frac{3}{10} + \frac{1}{6}$

15. Find the following products. Use shortcuts. Express each answer in simplest form.

    a. $\frac{2}{3} \times \frac{4}{5} \times \frac{5}{7} \times \frac{1}{8}$

    b. $1\frac{1}{8} \times 3\frac{3}{4} \times 1\frac{1}{3} \times 1\frac{3}{5}$

Space for Your Work

Space for Your Work

16. Simplify each of the following. Express each answer in simplest form.

    a. $\frac{1}{2} + \frac{3}{4} \times \frac{5}{9}$

    b. $\frac{2}{3} \times \frac{3}{4} - \frac{1}{2} \times \frac{2}{3}$

    c. $\left(1\frac{1}{3} + \frac{7}{9}\right) \div \frac{2}{9}$

17. Solve the following problems. You may use a calculator if you wish, but give accurate answers. Express each answer in simplest form.

    a. Shelley watched television for $\frac{3}{4}$ h and she studied for $\frac{1}{3}$ h. How much time did she spend doing the two activities?

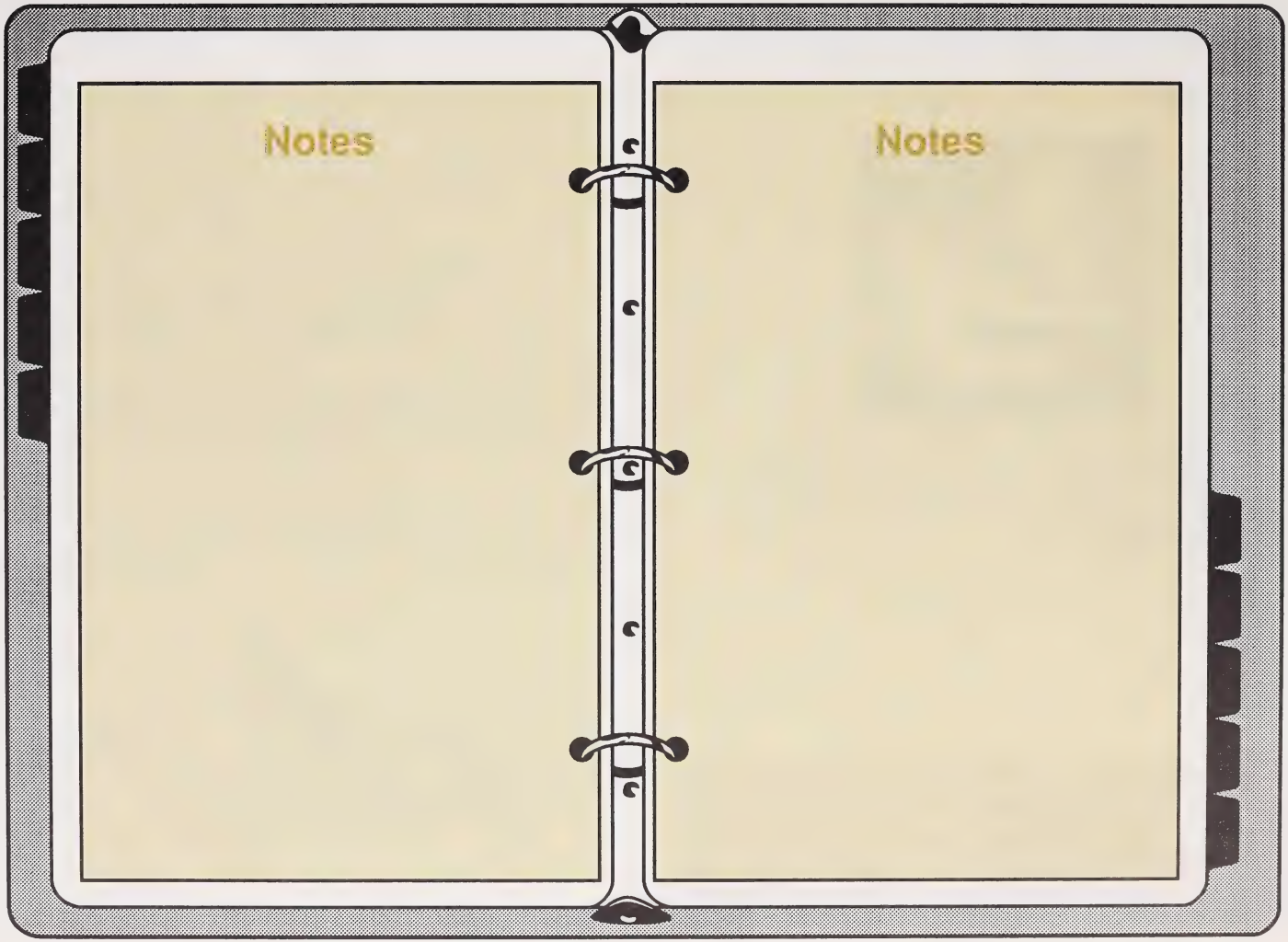
    b. It takes John $1\frac{3}{4}$ minutes to swim a length of the pool. How many lengths can he swim in 14 minutes?

c. An iceberg has a volume of 200 m^3. If $\frac{1}{8}$ of the iceberg is above the surface of the water, how many cubic metres of the iceberg are above the surface?

d. Alpha Centauri is $4\frac{1}{3}$ light years away from Earth. Sirius is $8\frac{3}{5}$ light years away from Earth. How much further from Earth is Sirius than Alpha Centauri?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.


Notes
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What Lies Ahead

In this section you will learn these skills.

- interpreting a fraction
- writing equivalent fractions
- writing basic fractions
- writing mixed numbers as improper fractions
- writing improper fractions as mixed numbers

In this section you will use these words.

- fraction
- numerator
- denominator
- equivalent fraction
- basic fraction
- simplest form

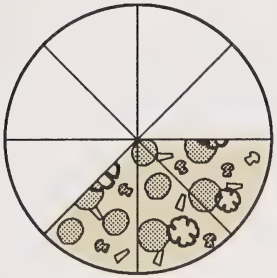
Working Together

In everyday situations you do not always encounter whole quantities. Often you require parts of a whole.

One way to express a part of a whole is to use a fraction.

Example 1

What fraction of this whole pizza is left?



The number of pieces left is 3.
The total number of pieces was 8.

So, $\frac{3}{8}$ of the pizza is left.

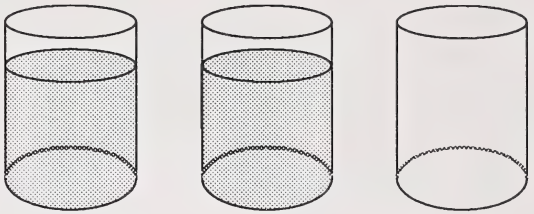
Read this fraction as three-eighths.

Note

$\frac{3}{8}$ ← numerator
← denominator

Example 2

What fraction of the glasses does not have juice?



The number of glasses without juice is 1.
The total number of glasses is 3.

So, $\frac{1}{3}$ of the glasses do not have juice.

Read this fraction as one-third.

Note

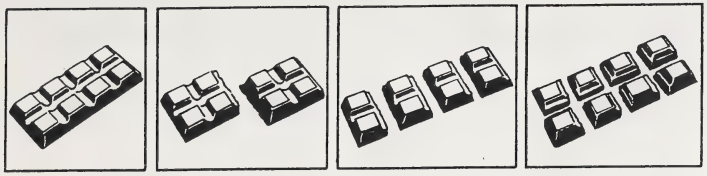
$\frac{1}{3}$ ← numerator
← denominator

Equivalent Forms of a Fraction

Fractions have many different names.

Example 1

These fractions name the same part of a whole chocolate bar.



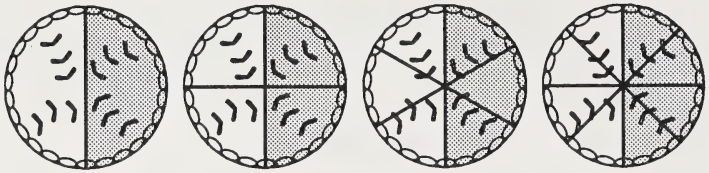
1 $\frac{2}{2}$ $\frac{4}{4}$ $\frac{8}{8}$

$$1 = \frac{2}{2} = \frac{4}{4} = \frac{8}{8}$$

When fractions name the same part of a whole, they are said to be **equivalent fractions**.

Example 2

Mrs. Koesling made four pies and cut them in different ways. Lorraine ate $\frac{1}{2}$ of a pie, Cory ate $\frac{2}{4}$ of a pie, Dwight ate $\frac{3}{6}$ of a pie, and Jocelyn ate $\frac{4}{8}$ of a pie.



Lorraine's share | Cory's share | Dwight's share | Jocelyn's share

From the diagram you can see that each person ate the same amount of a whole pie.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

These fractions are equivalent fractions.

Identity Property

From previous work you will recall that you may multiply or divide a whole number by 1 without changing its value. Here are some examples.

$$2 \times 1 = 2$$

$$4 \div 1 = 4$$

This is called the **identity property**.

The identity property applies to fractions as well as to whole numbers. See these examples.

$$\frac{3}{4} \times 1 = \frac{3}{4}$$

$$\frac{1}{2} \div 1 = \frac{1}{2}$$

Note

There are many different forms of 1.

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5}$$

The identity property helps you to write equivalent fractions. See these examples.

$$\frac{1}{2} \times \boxed{1} \; \boxed{\frac{2}{2}} = \frac{2}{4}$$

$$\frac{6}{12} \div \boxed{1} \; \boxed{\frac{3}{3}} = \frac{2}{4}$$

You can write equivalent fractions by multiplying the numerator and the denominator by the same number. Here is an example.

Write three equivalent fractions for $\frac{1}{2}$.

$$\frac{1}{2} \overset{\times 2}{=} \frac{2}{4} \qquad \frac{1}{2} \overset{\times 3}{=} \frac{3}{6} \qquad \frac{1}{2} \overset{\times 4}{=} \frac{4}{8}$$

You may write equivalent fractions by dividing the numerator and the denominator by the same number. See the following example.

Write three equivalent fractions for $\frac{6}{12}$.

$$\frac{6}{12} \overset{\div 2}{=} \frac{3}{6} \qquad \frac{6}{12} \overset{\div 3}{=} \frac{2}{4} \qquad \frac{6}{12} \overset{\div 6}{=} \frac{1}{2}$$

Basic Fractions

A basic fraction is a fraction with the lowest possible denominator. It is written in **simplest form**.

The following are basic fractions.

$\frac{1}{3}, \frac{2}{5}, \frac{7}{8}, \frac{9}{10}$

The following are not basic fractions.

$\frac{4}{12}, \frac{6}{15}, \frac{14}{16}, \frac{45}{60}$

Example

Write the basic fraction for $\frac{12}{90}$.

Method 1

The basic fraction can be found by dividing the numerator and the denominator by the greatest common factor (GCF) of the two numbers.

The GCF of 12 and 90 is 6.

$\frac{12}{90} \overset{\div 6}{=} \frac{2}{15}$

2 and 15 are the smallest possible whole number numerator and denominator of $\frac{12}{90}$.

$\frac{2}{15}$ is the simplest form of $\frac{12}{90}$.

$\frac{2}{15}$ is a basic fraction.

Method 2

Here is another way to find a basic fraction.

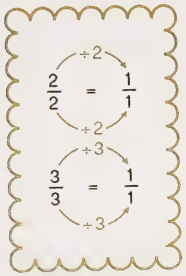
- Write the numerator and the denominator as a product of prime factors.
- Cancel the common factors.
- Multiply the remaining factors.

$$\frac{12}{90} = \frac{2 \times 2 \times 3}{2 \times 3 \times 3 \times 5}$$

$$= \frac{\cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times 3 \times \cancel{3} \times 5}$$

$$= \frac{1 \times 2 \times 1}{1 \times 3 \times 1 \times 5}$$

$$= \frac{2}{15}$$



So, $\frac{2}{15}$ is the basic fraction of $\frac{12}{90}$.

When you use the prime factor method for finding the basic fraction, you will discover that common factors are not always directly above each other. No matter where the common factors are, they can be cancelled as long as one is in the numerator and the other is in the denominator.

$$\frac{12}{90} = \frac{2 \times \cancel{2} \times \cancel{3}}{\cancel{2} \times 3 \times \cancel{3} \times 5}$$

$$= \frac{2}{15}$$

Cancelling may also be done in this way,

$$\begin{array}{r} 2 \\ \cancel{12} \\ \cancel{90} \\ 15 \end{array}$$

Divide 12 and 90 by 6.

or in this way.

$$\begin{array}{r} 2 \\ \cancel{6} \\ \cancel{12} \\ \cancel{90} \\ \cancel{45} \\ 15 \end{array}$$

Divide 12 and 90 by 2 and then divide 6 and 45 by 3.

So, $\frac{12}{90} = \frac{2}{15}$.

Mixed Numbers and Improper Fractions

Fractions greater than 1 can be expressed as a mixed number or as an improper fraction.

A **mixed number** has a whole number component and a fraction component all in one value.

An **improper fraction** is a fraction in which the numerator is greater than the denominator.

Example 1

Jason ran $4\frac{1}{3}$ laps around the track. Express $4\frac{1}{3}$ as an improper fraction.

Method 1

Here is one way to find the improper fraction.

$$4\tfrac{1}{3} = 1 + 1 + 1 + 1 + \tfrac{1}{3}$$

$$= \tfrac{3}{3} + \tfrac{3}{3} + \tfrac{3}{3} + \tfrac{3}{3} + \tfrac{1}{3}$$

$$= \tfrac{12}{3} + \tfrac{1}{3}$$

$$= \tfrac{13}{3}$$

$4\frac{1}{3}$ is a mixed number and $\frac{13}{3}$ is the improper fraction for this mixed number.

$$4\tfrac{1}{3} = \tfrac{13}{3}$$

Method 2

Here is a quick way to find the improper fraction.

- Multiply the denominator of the fraction by the whole number. Next add the numerator of the fraction part to this product.
- Write the resulting number over the denominator of the original fraction.

$$4\tfrac{1}{3} = \frac{4 \times 3 + 1}{3}$$

$$= \tfrac{13}{3}$$

Example 2

Joseph has $\frac{19}{6}$ boxes of ornaments. Express $\frac{19}{6}$ as a mixed number.

Method 1

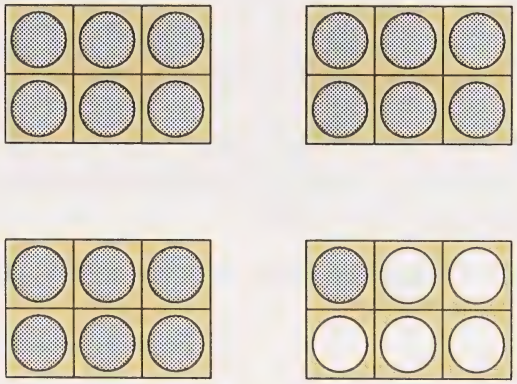
$$\frac{19}{6} = \frac{6}{6} + \frac{6}{6} + \frac{6}{6} + \frac{1}{6}$$

$$= 1 + 1 + 1 + \frac{1}{6}$$

$$= 3\frac{1}{6}$$

So, $\frac{19}{6} = 3\frac{1}{6}$.

This diagram may help you to see the situation.



Method 2

Here is a quick way to find the mixed form.

- Divide the numerator by the denominator.
- Write the whole number part of the quotient.
- Then divide the remainder by the denominator and write this part of the quotient as a fraction.

$\frac{19}{6}$ means 19 ÷ 6.

$$\begin{array}{r} 3 \\ 6\overline{)19} \\ \underline{18} \\ 1 \end{array}$$

← 3 wholes

← The remainder of 1 is divided by 6, too.

So, $\frac{19}{6} = 3\ \frac{\text{remainder}}{6}$

$= 3\frac{1}{6}$.

Practice Activities

Space for Your Work

1. Label each part of the fraction. What is Part a called? What is Part b called?

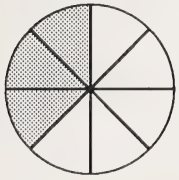
$$\frac{6}{10} \quad \begin{matrix} \leftarrow \text{a} \\ \leftarrow \text{b} \end{matrix}$$

2. a. What does Part a in Question 1 show?

   b. What does Part b in Question 1 show?

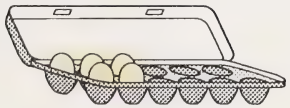
3. For each of the following diagrams, write a fraction to describe the part of the whole being represented.

a.

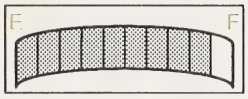


b.

c.



d.



4. Find three equivalent fractions for each of the following basic fractions.

Space for Your Work

a. $\frac{1}{2}$

b. $\frac{2}{3}$

c. $\frac{3}{5}$

d. $\frac{7}{10}$

5. Find the basic fraction for each of these unsimplified forms.

a. $\frac{30}{45}$

b. $\frac{16}{40}$

c. $\frac{75}{100}$

d. $\frac{12}{36}$

6. Express each of these improper fractions as a mixed number. Make sure that the fraction part is expressed as a basic fraction.

   a. $\frac{45}{30}$

   b. $\frac{40}{16}$

   c. $\frac{100}{75}$

   d. $\frac{36}{12}$

7. Express each of these improper fractions as a mixed number.

   a. $\frac{10}{3}$

   b. $\frac{16}{7}$

   c. $\frac{21}{4}$

   d. $\frac{33}{2}$

Space for Your Work

8. Express each of these mixed numbers as an improper fraction.

Space for Your Work

a. $5\frac{1}{4}$

b. $3\frac{2}{5}$

c. $2\frac{1}{3}$

d. $4\frac{3}{5}$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Number lines are useful for finding equivalent and basic fractions. You can make number lines using two sheets of plain paper. Be sure the sheets of paper are the same size.

First, using a straightedge, draw three lines on one sheet of paper.

Next, fold the paper in half. Unfold the paper and label the first number line as shown.

$\frac{1}{2}$	

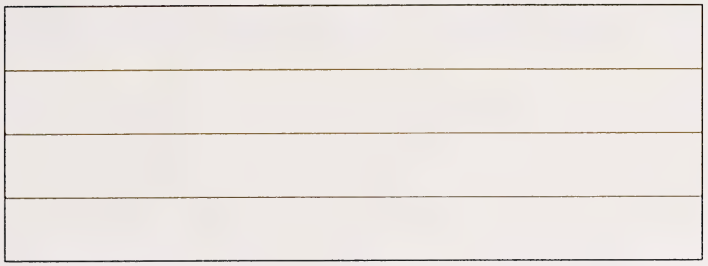
Now fold the paper again. Then fold it in half a second time. Unfold the paper and label the second line as shown.

		$\frac{1}{2}$		
	$\frac{1}{4}$		$\frac{2}{4}$	$\frac{3}{4}$

Now refold the paper again. Then fold it in half a third time. Unfold the paper and label the third line as shown.

			$\frac{1}{2}$				
		$\frac{1}{4}$		$\frac{2}{4}$		$\frac{3}{4}$	
$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	

Now, using a straightedge, draw three lines on the second sheet of paper.

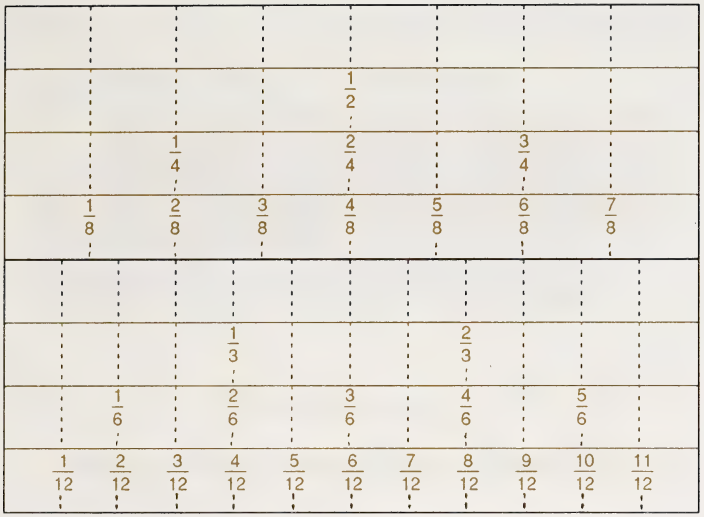


Then fold the paper into thirds. This is a bit tricky. Unfold the paper and label the first line as shown.

Now refold the paper again. Next, fold it in half again. Unfold the paper and label the second line as shown.

Now refold the paper again. For the last time, fold it in half again. Unfold the paper and label the third line as shown.

Finally tape the two sheets together.



Example 1: Find two fractions that are equivalent to $\frac{3}{4}$.

Solution

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$$

They are all on the same fold.

Example 2: Find the basic fraction of $\frac{3}{12}$.

Solution

$$\frac{3}{12} = \frac{1}{4}$$

They are both on the same fold and $\frac{1}{4}$ has the smallest possible denominator.

Multiple bars are useful for finding equivalent fractions. You can make multiple bars using popsicle sticks, tongue depressors, or strips of paper.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Example 1: Find the equivalent fractions for $\frac{1}{4}$.

Solution: Use the 1 and 4 bars.

1	2	3	4	5	6	7	8	9	10
4	8	12	16	20	24	28	32	36	40

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} = \frac{7}{28} = \frac{8}{32} = \frac{9}{36} = \frac{10}{40}$$

Example 2: Find the basic fraction of $\frac{18}{45}$.

Solution: 18 and 45 are aligned on the 2 and 5 bars.

2	4	6	8	10	12	14	16	**18**	20
5	10	15	20	25	30	35	40	**45**	50

So, $\frac{18}{45} = \frac{2}{5}$.

$\frac{2}{5}$ is the basic fraction of $\frac{18}{45}$.

Example 3: Find the basic fraction of $\frac{24}{36}$.

Method 1

24 and 36 are aligned on the 4 and 6 bars.

4	8	12	16	20	**24**	28	32	36	40
6	12	18	24	30	**36**	42	48	54	60

So, $\frac{24}{36} = \frac{4}{6}$.

But, this is not the basic fraction as 4 and 6 are also aligned on the 2 and 3 bars.

2	**4**	6	8	10	12	14	16	18	20
3	**6**	9	12	15	18	21	24	27	30

So, $\frac{24}{36} = \frac{4}{6} = \frac{2}{3}$.

$\frac{2}{3}$ is the basic fraction of $\frac{24}{36}$.

Method 2

24 and 36 are aligned on the 6 and 9 bars.

6	12	18	**24**	30	36	42	48	54	60
9	18	27	**36**	45	54	63	72	81	90

So, $\frac{24}{36} = \frac{6}{9}$.

But, this is not the basic fraction as 6 and 9 are also aligned on the 2 and 3 bars.

2	4	**6**	8	10	12	14	16	18	20
3	6	**9**	12	15	18	21	24	27	30

So, $\frac{24}{36} = \frac{6}{9} = \frac{2}{3}$.

$\frac{2}{3}$ is the basic fraction of $\frac{24}{36}$.

Extra Practice

Space for Your Work

Make number lines and multiple bars like those on the previous pages. Use these number lines and multiple bars to answer Questions 1 and 2.

1. Use the number lines or the multiple bars to find two equivalent fractions for each of the following.

   a. $\frac{1}{6}$

   b. $\frac{3}{4}$

   c. $\frac{7}{8}$

   d. $\frac{1}{9}$

2. Use number lines or multiple bars to find the basic fraction for each of the following.

   a. $\frac{6}{12}$

   b. $\frac{4}{8}$

   c. $\frac{20}{35}$

   d. $\frac{48}{64}$

3. Rewrite each of the following improper fractions as a mixed number.

Space for Your Work

   a. $\frac{77}{8}$

   b. $\frac{31}{4}$

   c. $\frac{36}{7}$

   d. $\frac{23}{5}$

4. Rewrite each of the following mixed numbers as an improper fraction.

   a. $2\frac{3}{5}$

   b. $3\frac{7}{8}$

   c. $1\frac{4}{9}$

   d. $2\frac{3}{10}$

Computer Alternative

Space for Your Work

5. Do Lessons 2, 3, and 4 of the Fraction disk from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA). If you need help or make errors, remember to hold down the SHIFT key and press the [?] key.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Here is how to use a calculator to change an improper fraction to a mixed number.

Step 1: Divide the numerator by the denominator.

Step 2: Subtract the whole number part from the quotient.

Step 3: Multiply what is left by the denominator of the fraction.

Step 4: Use this number as the numerator of the fraction part of the mixed number.

Example: Express $\frac{21}{8}$ as a mixed number.

Key Press	Display	
21 ÷ 8 =	2.65	← The whole number part is 2.
− 2 =	0.65	
× 8 =	5	← The numerator of the fraction part is 5.

The mixed number for $\frac{21}{8}$ is $2\frac{5}{8}$.

Note: Both the improper fraction form and the mixed number form have the same denominator.

Here is how to use a calculator to change a mixed number to a fraction.

Step 1: Multiply the whole number part and the denominator.

Step 2: Add the numerator.

Step 3: Express this amount over the original denominator of the fraction part.

Example: Express $3\frac{7}{8}$ as an improper fraction.

Key Press	Display
3 × 8 =	24
+ 7 =	31

The numerator of the improper fraction is 31.

The improper fraction for $3\frac{7}{8}$ is $\frac{31}{8}$.

Concluding Activities

Space for Your Work

1. Use a calculator to write the mixed number for each of these improper fractions.

   a. $\frac{125}{8}$

   b. $\frac{87}{5}$

   c. $\frac{77}{2}$

   d. $\frac{215}{7}$

   e. $\frac{312}{9}$

2. Use a calculator to change each of these mixed numbers to an improper fraction.

   a. $7\frac{7}{12}$

   b. $4\frac{9}{16}$

   c. $3\frac{24}{25}$

   d. $2\frac{33}{64}$

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

- expressing a fraction as a decimal number
- expressing a terminating decimal number as a fraction

In this section you will learn these terms.

- terminating decimal numbers
- repeating decimal numbers

Working Together

A person often has many different names. For example, a child may call a woman *Mother*. A husband may call the same woman *Betty*. A nephew may call the same lady *Auntie*.

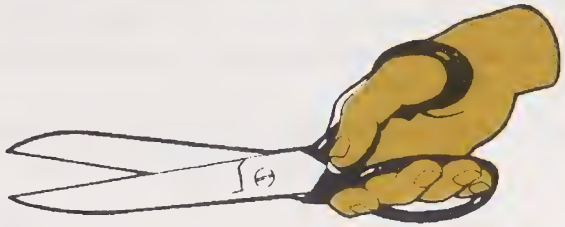
Numbers also have many different names. In this section you will learn to change a number from its fraction form to its decimal number form and from its decimal number form to its fraction form.

Expressing a Decimal Number as a Fraction

Sometimes you want to change a decimal number to its fraction form.

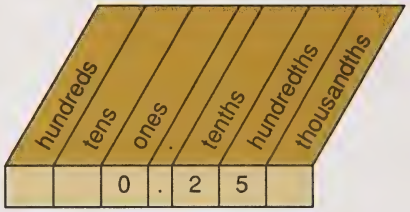
Example 1

Anita took 0.25 h to cut out the pieces of material for her dress. Express this time as a fraction.



Method 1

Use a place value chart to help you.



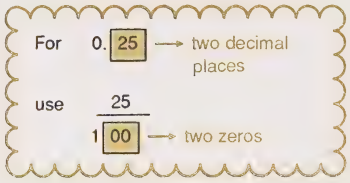
0.25 means 25 hundredths or $\frac{25}{100}$.

So, $0.25 = \frac{25}{100}$ or $\frac{1}{4}$.

Anita took $\frac{1}{4}$ h to cut out the pieces of material for her dress.

Method 2

Use this memory tool.



So, $0.25 = \frac{25}{100}$ or $\frac{1}{4}$.

Anita took $\frac{1}{4}$ h to cut out the pieces of material for her dress.

Expressing a Fraction as a Decimal Number

Sometimes you want to change a fraction to a decimal number.

Example 1

Polly the parrot eats $\frac{1}{2}$ a cracker every day. Express this fraction as a decimal number.

Method 1

Here is one way to change $\frac{1}{2}$ to a decimal number.

Then use this memory tool.

For $\frac{5}{10}$ → one zero

Use 0.5 → one decimal place

So, $\frac{1}{2} = 0.5$.

Method 2

Here is another way to change $\frac{1}{2}$ to a decimal number.

$\frac{1}{2}$ can mean 1 ÷ 2.

$$\begin{array}{r} 0.5 \\ 2\overline{)1.0} \\ \underline{1.0} \\ 0 \end{array}$$

So, $\frac{1}{2} = 0.5$.

A calculator can be used to find the equivalent decimal number.

Key Press	Display
1 ÷ 2 =	0.5

Polly the parrot eats 0.5 crackers every day.

Note

Decimal numbers which end with a non-repeating digit other than zero are called **terminating decimal numbers**.

Example 2

Priscilla the parrot eats $\frac{2}{3}$ of an apple every day.
Express the fraction as a decimal number.

Method 1

$\frac{2}{3}$ can mean $2 \div 3$.

$$
\begin{array}{r}
0.666 \\
3\overline{)2.000} \\
\underline{18} \\
20 \\
\underline{18} \\
20 \\
\underline{18} \\
2
\end{array}
$$

The only remainders are 2s.

When dividing by 3 the two possible remainders are 1 and 2. So the maximum number of digits possible in the repeating block is two.

Since the only remainder that occurs is 2, there is only one digit in the repeating block. This can be shown using dot notation.

$\frac{2}{3} = 0.\dot{6}$ ← one digit in the repeating block

Priscilla the parrot eats $0.\dot{6}$ apples every day.

Method 2

A calculator can be used to find the equivalent decimal number.

Key Press	Display
2 ÷ 3 =	0.6666666

Priscilla the parrot eats $0.\dot{6}$ apples every day.

Note

Decimal numbers which continue with a repeating pattern of digits are called **repeating decimals**.

It is often more convenient to round off repeating decimals.

$0.666... \doteq 0.7$

Priscilla the parrot eats about 0.7 apples every day.

Example 3

Jessie has hockey practice on $\frac{2}{7}$ of the afternoons in one week. Express the fraction as a decimal number.

Method 1

$\frac{2}{7}$ can mean $2 \div 7$.

$$
\begin{array}{r}
0.285\,714\,2 \\
7\overline{)\,2.000\,000\,0} \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
50 \\
49 \\
\hline
10 \\
7 \\
\hline
30 \\
28 \\
\hline
20 \\
14 \\
\hline
6
\end{array}
$$

There are six different remainders.

The remainders start to repeat.

When dividing by 7, the six possible remainders are 1, 2, 3, 4, 5, and 6. So, the maximum number of digits in the repeating block is six. Since each remainder occurred, there are six digits in the repeating block.

$$\frac{2}{7} = 0.\dot{2}85\,714\dot{}$$

It may be more convenient to round off the repeating decimal number.

$$\frac{2}{7} \doteq 0.3$$

Jessie has hockey practice on about 0.3 of the afternoons in each week.

Method 2

A calculator can be used to find the equivalent decimal number for $\frac{2}{7}$.

Key Press	Display
2 ÷ 7 =	0.2857142

$$\frac{2}{7} = 0.\dot{2}85\,714\dot{}$$

$$\frac{2}{7} \doteq 0.3$$

Jessie has hockey practice on about 0.3 of the afternoons in each week.

Example 4

Jackson hit the baseball safely $\frac{5}{6}$ of the times that he was up to bat. Express the fraction as a decimal number.

Solution

$\frac{5}{6}$ can mean $5 \div 6$.

$$
\begin{array}{r}
0.833 \\
6\overline{)5.000} \\
48 \\
\hline
20 \\
18 \\
\hline
20 \\
18 \\
\hline
2
\end{array}
$$

The only remainders are 2s.

When dividing by 6, there are five possible remainders which are 1, 2, 3, 4, and 5. This means that the maximum number of digits in the repeating block is five. Since only the remainder 2 occurs, there is only one digit in the repeating block.

$$\frac{5}{6} = 0.8\dot{3}$$

or $\frac{5}{6} \doteq 0.8$

Jackson hit safely about 0.8 of the times he was up to bat.

Sometimes only one digit repeats, as in Examples 2 and 4. On other occasions a block of digits repeats, as in Example 3.

Example 5

Express each of the following using dot notation.

- 0.444...
- 0.7272...
- 0.3111...
- 0.145 858...

Solution

- $0.444... = 0.\dot{4}$
- $0.7272... = 0.\dot{7}\dot{2}$
- $0.3111... = 0.3\dot{1}$
- $0.145\,858... = 0.14\dot{5}\dot{8}$

Note

Some textbooks use bar notation instead of dot notation. The above solutions can also be shown as follows.

- $0.444... = 0.\overline{4}$
- $0.7272... = 0.\overline{72}$
- $0.3111... = 0.3\overline{1}$
- $0.145\,858... = 0.14\overline{58}$

Practice Activities

Space for Your Work

1. Express each of these decimal numbers as a fraction in simplest form.

   a. 0.875

   b. 0.3

   c. 0.26

   d. 0.036

   e. 0.05

2. Express each of these fractions as a terminating decimal number.

   a. $\frac{1}{8}$

   b. $\frac{23}{25}$

   c. $\frac{3}{4}$

   d. $\frac{3}{125}$

3. If each if these fractions is expressed as a repeating decimal number, what is the maximum number of digits in the repeating block?

   a. $\frac{7}{9}$

   b. $\frac{5}{12}$

   c. $\frac{9}{11}$

   d. $\frac{2}{13}$

4. Write each of the fractions in Question 3 as repeating decimal numbers. Use the dot or bar notation.

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Some fractions will result in terminating decimal numbers. Here are some examples.

   a. $\frac{1}{8} = 0.875$

   b. $\frac{3}{20} = 0.15$

   c. $\frac{2}{5} = 0.4$

   d. $\frac{1}{50} = 0.02$

   Express the denominator of each fraction as a product of prime factors.

2. Some fractions will result in repeating decimal numbers. Here are some examples.

   a. $\frac{9}{11} = 0.\dot{8}\dot{1}$

   b. $\frac{7}{12} = 0.583\dot{3}$

   c. $\frac{5}{18} = 0.2\dot{7}$

   d. $\frac{2}{13} = 0.\dot{1}53\,846\dot{}$

   Express the denominator of each fraction as a product of prime factors.

3. a. Compare the prime factors in Questions 1 and 2. What do you notice?

   b. State a rule which can be used to determine if a fraction will result in a terminating decimal number.

4. Use the rule you discovered in Question 3 to determine if each of the following fractions will result in a terminating or repeating decimal number.

   a. $\frac{11}{12}$

   b. $\frac{1}{16}$

   c. $\frac{8}{125}$

   d. $\frac{9}{40}$

   e. $\frac{10}{49}$

5. a. Look at the prime factors in Question 1. Compare the number of 2s and the number of 5s to the number of decimal places. What do you notice?

   b. State a rule which can be used to determine the number of decimal places in the terminating decimal that will result when a fraction is changed to a decimal number.

Space for Your Work

6. Use the rule you discovered in Question 5 to decide how many decimal places there will be when each of these fractions is changed to a terminating decimal number.

   a. $\frac{4}{25}$

   b. $\frac{5}{8}$

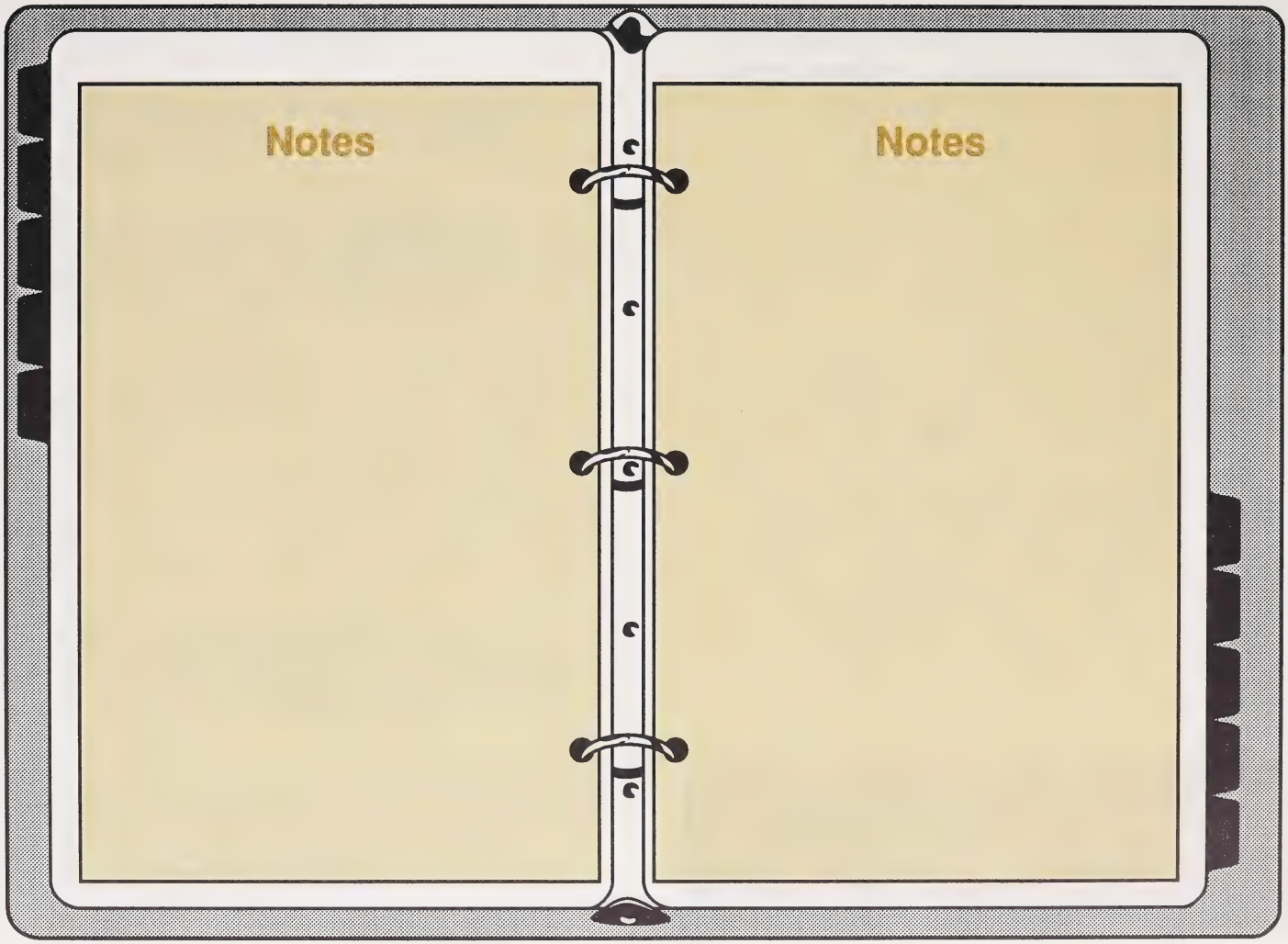
   c. $\frac{3}{50}$

   d. $\frac{1}{40}$

   e. $\frac{1}{200}$

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will review these skills.

- comparing fractions with common numerators
- comparing fractions with common denominators
- comparing fractions with different numerators and different denominators

In this section you will learn these terms.

- is greater than
- is less than
- cross products

Working Together

In this section you will be comparing fractions.

Remember these symbols.

$<$ is less than

$>$ is greater than

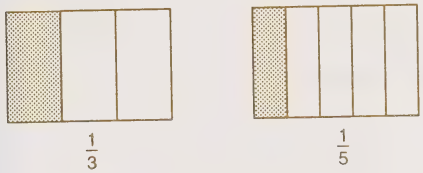
$=$ equals or is equal to

It is easy to compare fractions with common denominators or common numerators. It is more difficult to compare fractions with different denominators and different numerators.

Example 1: Common Numerators

Joan ate $\frac{1}{3}$ of a cake. Bill ate $\frac{1}{5}$ of a cake. If the two cakes were the same size, which of the two people ate more cake?

Solution



From the diagram it is easy to see that $\frac{1}{3}$ is larger than $\frac{1}{5}$.

So, $\frac{1}{3} > \frac{1}{5}$.

This is read as "is greater than".

Joan ate more cake.

Example 2: Common Denominators

Adam ate $\frac{1}{3}$ of a pie. Anna ate $\frac{2}{3}$ of a pie. If the two pies were the same size, which of these two people ate less pie?

Solution

From the diagram it is easy to see that $\frac{1}{3}$ is less than $\frac{2}{3}$.

So, $\frac{2}{3} > \frac{1}{3}$.

This is read as "is less than".

Anna ate less pie.

Example 3: Different Numerators and Different Denominators

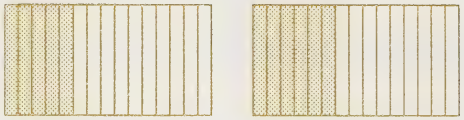
Bill ate $\frac{1}{3}$ of a chocolate bar. Belinda ate $\frac{2}{5}$ of a chocolate bar. If the chocolate bars were the same size, which person ate a larger portion of the chocolate bar?

Solution

$\frac{1}{3}$ $\frac{2}{5}$

This time it is hard to tell from the diagram which person ate a greater portion of the chocolate bar.

One way to make it easier to compare is to cut both chocolate bars into pieces that are the same size. By doing this you would be renaming the fractions with common denominators.



$\frac{5}{15}$ $\frac{6}{15}$

From the new diagram it is easy to see that $\frac{6}{15}$ is larger than $\frac{5}{15}$.

$$\frac{6}{15} > \frac{5}{15}$$

So, $\frac{2}{5} > \frac{1}{3}$.

Belinda ate a larger portion of the chocolate bar.

When comparing two or more fractions without objects or diagrams, you should examine the numerators and denominators.

Example 1: Which is larger, $\frac{2}{5}$ or $\frac{2}{7}$?

Solution

The fractions have common numerators.

2 fifths > 2 sevenths

So, $\frac{2}{5} > \frac{2}{7}$.

Example 2: Which is larger, $\frac{2}{5}$ or $\frac{4}{5}$?

Solution

The fractions have common denominators.

4 fifths > 2 fifths

So, $\frac{4}{5} > \frac{2}{5}$.

Example 3: Which is larger, $\frac{2}{3}$ or $\frac{4}{7}$?

Solution

The fractions have different numerators and denominators.

To compare these fractions, change them to equivalent fractions with common denominators.

The LCM of 3 and 7 is 21.

$\frac{2}{3} \overset{\times 7}{=} \frac{14}{21}$ $\quad$ $\frac{4}{7} \overset{\times 3}{=} \frac{12}{21}$

14 twenty-firsts > 12 twenty-firsts

$\frac{14}{21} > \frac{12}{21}$

So, $\frac{2}{3} > \frac{4}{7}$.

To compare two or more fractions you can also use equivalent decimal number forms. To find the decimal number form you may want to use a calculator.

Example 1: Which is larger, $\frac{3}{5}$ or $\frac{4}{7}$?

Solution

$\frac{3}{5} = 0.6$ $\quad\quad$ $\frac{4}{7} = 0.\dot{5}71\,428\dot{}$

Compare the digits in each place. Work from left to right.

$0 . 6$

↑ ↑

same different

↓ ↓

$0 . \dot{5}71\,428\dot{}$

$6 > 5$

So, $\frac{3}{5} > \frac{4}{7}$.

Example 2: Which is larger, $\frac{5}{7}$ or $\frac{3}{4}$?

Solution

$\frac{5}{7} = 0.\dot{7}14\,285\dot{}$ $\quad\quad$ $\frac{3}{4} = 0 . 75$

Compare the digits in each place. Work from left to right.

$0 . \dot{7}14\,285\dot{}$

↑ ↑↑

same different

↓ ↓↓

$0 . 75$

$5 > 1$

So, $\frac{3}{4} > \frac{5}{7}$.

Practice Activities

Space for Your Work

1. Use < or > to show which fraction is the larger of the two.

   a. $\frac{5}{8}$ ◯ $\frac{3}{8}$

   b. $\frac{9}{7}$ ◯ $\frac{11}{7}$

   c. $\frac{7}{10}$ ◯ $\frac{9}{10}$

2. Use < or > to show which fraction is the larger of the two.

   a. $\frac{4}{11}$ ◯ $\frac{4}{5}$

   b. $\frac{3}{8}$ ◯ $\frac{3}{10}$

   c. $\frac{1}{2}$ ◯ $\frac{1}{3}$

3. Use < or > to show which fraction is the larger of the two.

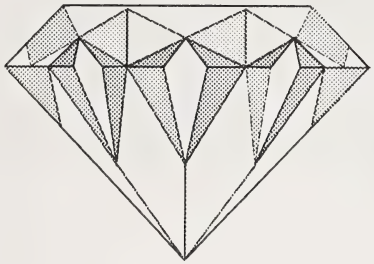
   a. $\frac{2}{5}$ ◯ $\frac{3}{7}$

   b. $\frac{7}{9}$ ◯ $\frac{2}{3}$

   c. $\frac{3}{4}$ ◯ $\frac{4}{5}$

4. If it took Marnee $\frac{1}{2}$ of an hour to do this section of the course and Darwin did it in $\frac{3}{4}$ of an hour, which person needed more time to complete the work?

5. Sue, Bill, and Micky bought diamonds at Jill's Jewellery. Sue bought a $\frac{1}{2}$ carat diamond, Bill bought a $\frac{5}{8}$ carat diamond, and Micky bought a $\frac{2}{3}$ carat diamond. Which of the three people bought the biggest diamond?



Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Earlier in this section you reviewed a method to compare fractions by finding equivalent fractions with common denominators and comparing the numerators.

A short cut method to compare fractions mentally is to use **cross products**. This method allows you to quickly compare the numerators of equivalent fractions with the same denominator.

Example: Which is less, $\frac{2}{5}$ or $\frac{3}{7}$?

Method 1

Find equivalent fractions.

The LCM for 5 and 7 is 35.

$$\frac{2}{5} \overset{\times 7}{=} \frac{14}{35} \qquad \frac{3}{7} \overset{\times 5}{=} \frac{15}{35}$$

Then compare the numerators.

$$14 < 15$$

So, $\frac{14}{35} < \frac{15}{35}$

and $\frac{2}{5} < \frac{3}{7}$.

Method 2

Find the cross products.

$2 \times 7 = 14$ $\qquad$ $3 \times 5 = 15$

$\frac{2}{5} \quad \frac{3}{7}$

Compare the cross products.

$$14 < 15$$

Since the numerator of $\frac{2}{5}$ was used to make the smaller cross product, $\frac{2}{5}$ is the smaller of the two fractions.

Concluding Activities

Space for Your Work

Use cross products to mentally compare each of the following pairs of fractions.

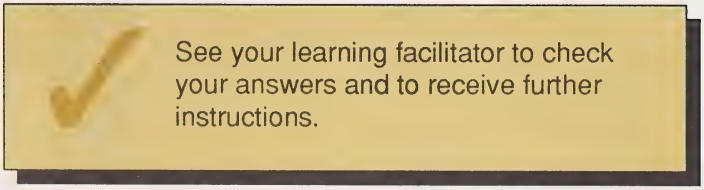
1. $\frac{4}{5}, \frac{7}{9}$

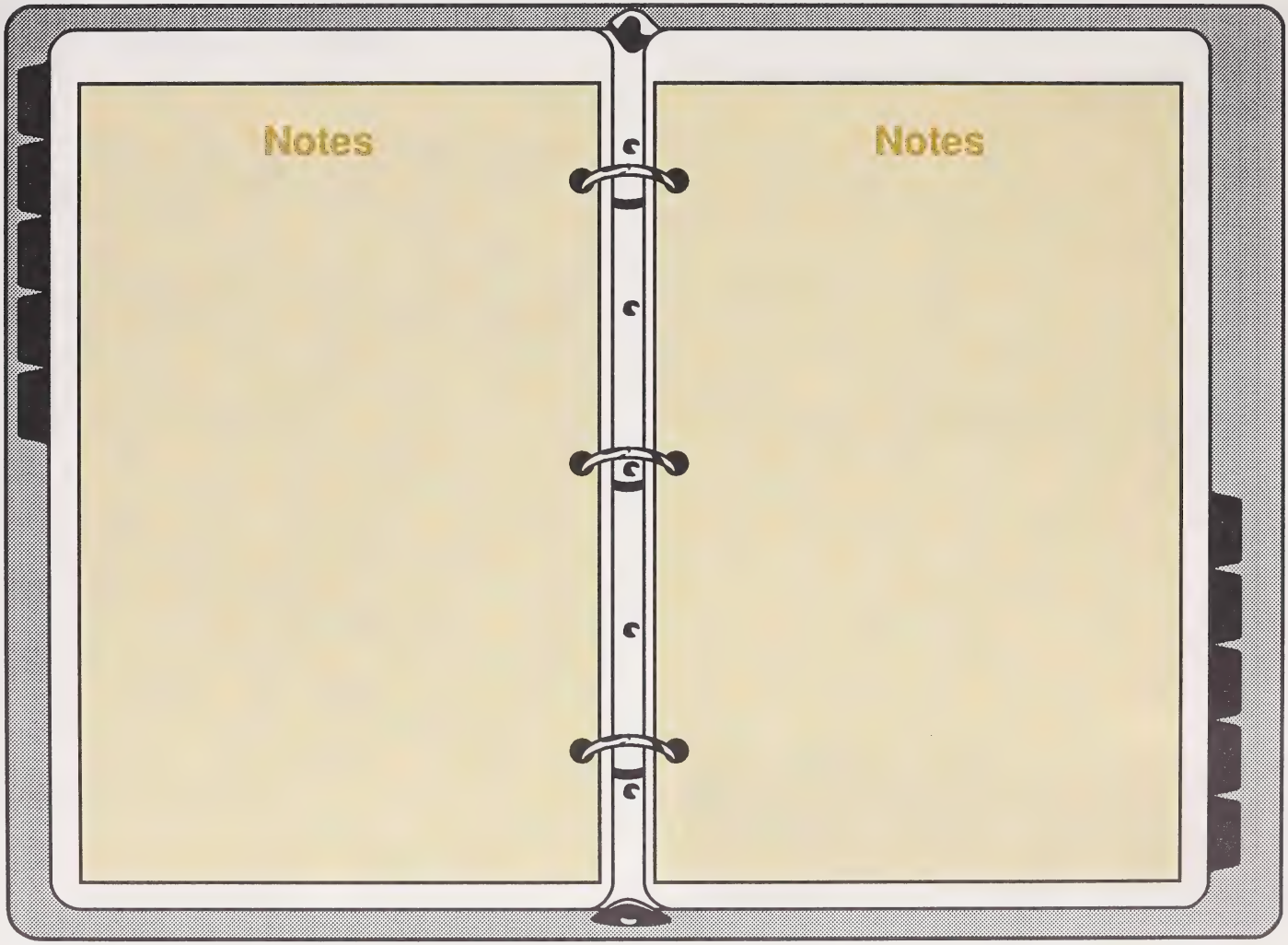
2. $\frac{7}{9}, \frac{6}{7}$

3. $\frac{2}{3}, \frac{6}{7}$

4. $\frac{4}{7}, \frac{1}{2}$

5. $\frac{2}{3}, \frac{4}{6}$




Notes
Notes

What Lies Ahead

In this section you will learn these skills.

- adding fractions and mixed numbers with like denominators
- subtracting fractions and mixed numbers with like denominators

In this section you will learn these terms.

- regrouping
- borrowing
- carrying

Working Together

Now you are ready to add and subtract fractions and mixed numbers. You will begin this section by adding and subtracting fractions and mixed numbers with like denominators.

Video Activity

Watch the video program *MATHWORKS: Adding and Subtracting Fractions and Mixed Numbers with Like Denominators* (Agency for Instructional Television). If you cannot watch the video simply read the notes that follow.

Example 1

Jack and Jill shared a medium pizza. Jack ate $\frac{3}{8}$ of the pizza and Jill ate $\frac{2}{8}$ of the pizza. What fraction of the entire pizza was eaten by Jack and Jill?

Solution

To add fractions with like denominators, add the numerators and keep the like denominator.

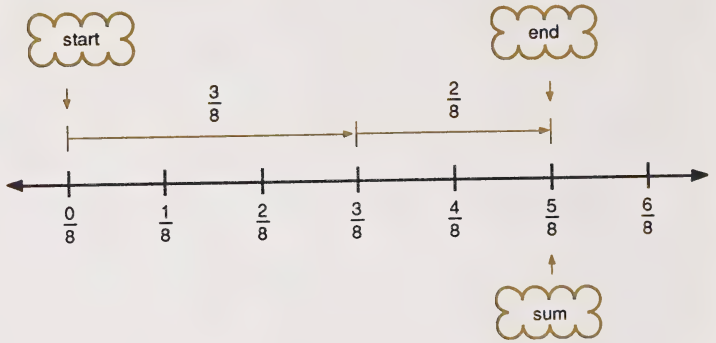
$$\frac{3}{8} + \frac{2}{8}$$

$$= \frac{3 + 2}{8} \quad \leftarrow \text{This step may be done mentally.}$$

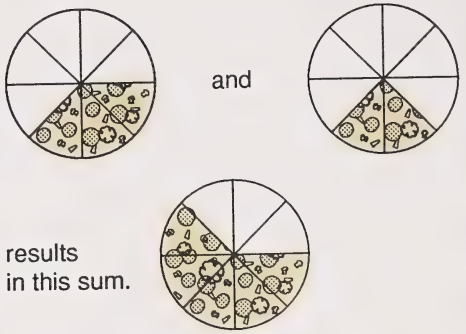
$$= \frac{5}{8}$$

Jack and Jill ate $\frac{5}{8}$ of the entire pizza.

A number line may help you understand the problem better.



A diagram can be used to help you visualize this situation.



Example 2

Jack and Jill began with $\frac{7}{8}$ of a pizza. After they ate $\frac{5}{8}$ of the pizza, what fraction of the whole pizza was left?

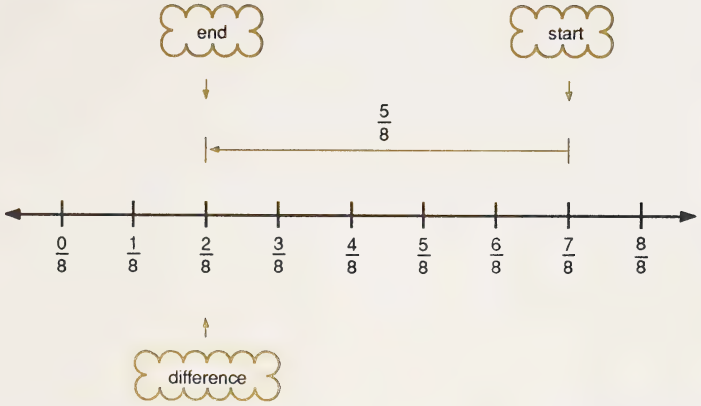
Solution

To subtract fractions with like denominators, subtract the numerators and keep the common denominator.

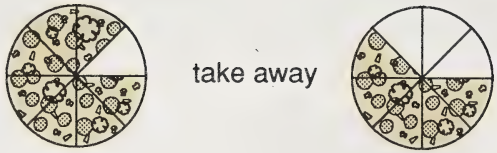
$$\frac{7}{8} - \frac{5}{8}$$
$$= \frac{7-5}{8}$$ ← This step can be done mentally.
$$= \frac{2}{8}$$
$$= \frac{1}{4}$$ ← Always express the final answer in simplest form.

So, $\frac{1}{4}$ of the whole pizza is left.

A number line may help you understand what happens.



A diagram can also be used to help you visualize the situation.

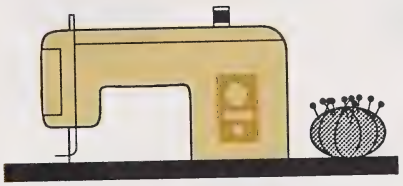


results in this difference.

The simplest form is this.

Example 3

Denzil sewed by hand for $1\frac{1}{4}$ hours. After that he sewed by machine for $2\frac{1}{4}$ hours. How long did he sew using both methods?



Method 1

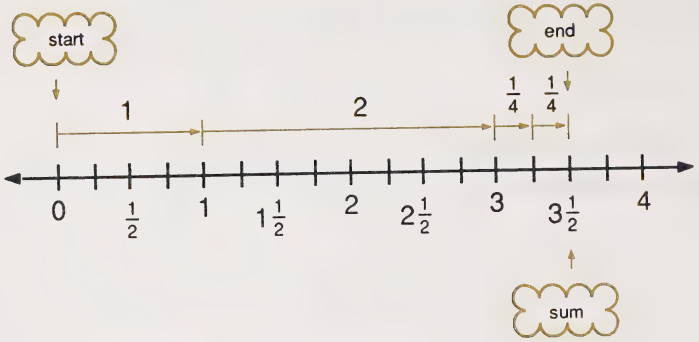
Work from right to left when adding vertically.

$$\begin{array}{r} 1\frac{1}{4} \\ +\ 2\frac{1}{4} \\ \hline 3\frac{2}{4} \end{array} = 3\frac{1}{2}$$

First add the fractions. Next add the whole numbers. Simplify the final answer.

Denzil sewed for $3\frac{1}{2}$ hours using both methods.

A number line can be used to help you visualize the total time spent sewing.



Method 2

Work from left to right when adding horizontally.

$$1\frac{1}{4} + 2\frac{1}{4}$$

$$= \frac{5}{4} + \frac{9}{4}$$

$$= \frac{14}{4}$$

$$= \frac{7}{2}$$

$$= 3\frac{1}{2}$$

First change mixed numbers to improper fractions. Then add the numerators and keep the like denominator.

Denzil sewed for $3\frac{1}{2}$ hours using both methods.

A number line can be used to help you visualize the time spent sewing.

Example 4

Jason had $3\frac{3}{4}$ loaves of bread. He uses $2\frac{1}{4}$ loaves to make sandwiches. How many loaves of bread does he have left?

Method 1

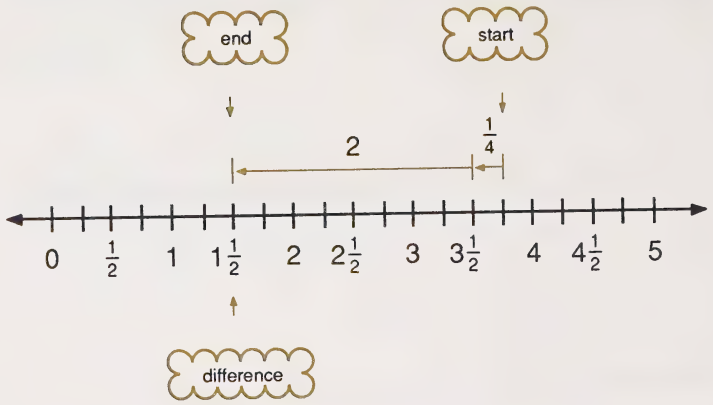
Work from right to left when subtracting vertically.

$$\begin{array}{r} 3\frac{3}{4} \\ -\ 2\frac{1}{4} \\ \hline 1\frac{2}{4} \end{array} = 1\frac{1}{2}$$

Subtract the fractions first. Next subtract the whole numbers. Simplify the final answer.

After making the sandwiches, Jason had $1\frac{1}{2}$ loaves of bread left.

A number line can also be used to help you visualize the situation.



Method 2

Work from left to right when subtracting horizontally.

$$3\tfrac{3}{4} - 2\tfrac{1}{4}$$

$$= \tfrac{15}{4} - \tfrac{9}{4}$$

$$= \tfrac{6}{4}$$

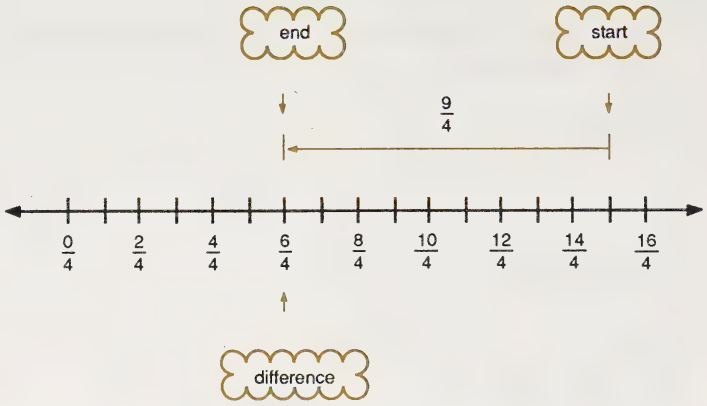
$$= 1\tfrac{2}{4}$$

$$= 1\tfrac{1}{2}$$

First change mixed numbers to improper fractions.

After making the sandwiches, Jason had $1\frac{1}{2}$ loaves of bread left.

A number line can also be used to visualize the situation.



Introductory Activities

Space for Your Work

1. Add each of the following. Express all final answers in simplest form.

   a. $\frac{2}{9} + \frac{6}{9}$

   b. $\frac{6}{11} + \frac{3}{11}$

   c. $\frac{3}{20} + \frac{13}{20}$

   d. $2\frac{2}{5} + 1\frac{1}{5}$

   e. $2\frac{1}{10} + 3\frac{7}{10}$

   f. $4\frac{5}{9} + 6\frac{1}{9}$

2. Subtract each of the following. Express all final answers in simplest form.

Space for Your Work

a. $\frac{7}{8} - \frac{5}{8}$

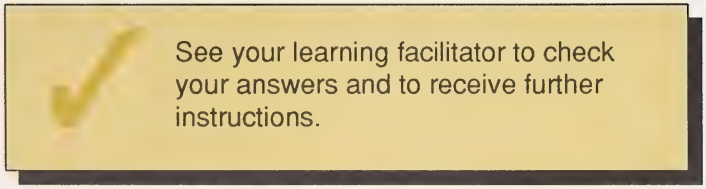
b. $\frac{4}{5} - \frac{1}{5}$

c. $\frac{6}{7} - \frac{3}{7}$

d. $1\frac{13}{100} - \frac{9}{100}$

e. $3\frac{5}{6} - 1\frac{2}{6}$

f. $16\frac{7}{9} - 4\frac{4}{9}$



See your learning facilitator to check your answers and to receive further instructions.

Working Together

Regrouping

Sometimes regrouping is needed to express an answer to an addition problem in simplest form.

Example 1

During the week Yvonne read $3\frac{1}{2}$ books, Nadine read $2\frac{1}{2}$ books, and Lucy read $1\frac{1}{2}$ books. In all how many books did the three girls read?

Solution

Two methods can be used to find the sum.

Method 1

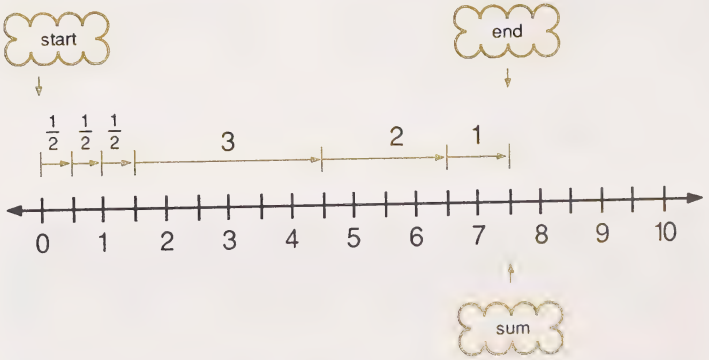
When adding vertically, work from right to left.

$$\begin{array}{r} 3\frac{1}{2} \\ 2\frac{1}{2} \\ +\ 1\frac{1}{2} \\ \hline 6\frac{3}{2} \end{array} = 6 + 1\frac{1}{2} = 7\frac{1}{2}$$

First add the fractions. Next add the whole numbers.

So, in all, the three girls read $7\frac{1}{2}$ books.

A number line can also be used to visualize the situation.



Method 2

When adding horizontally, work from left to right.

$$3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2}$$

$$= \frac{7}{2} + \frac{5}{2} + \frac{3}{2}$$

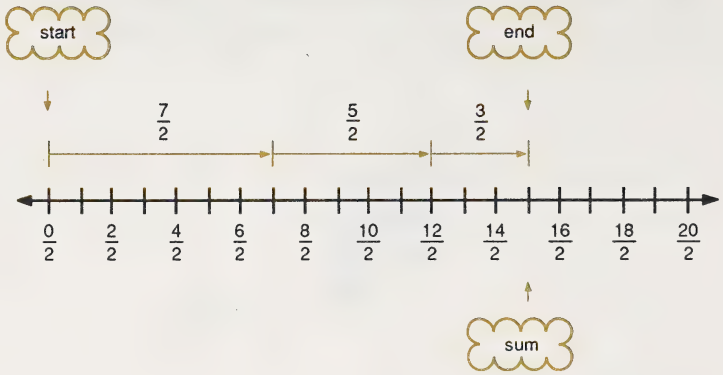
$$= \frac{15}{2}$$

$$= 7\frac{1}{2}$$

Change mixed numbers to improper fractions and then add.

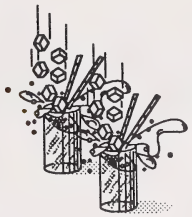
In all, the three girls read $7\frac{1}{2}$ books.

A number line can also be used to help you visualize the situation.



Example 2

Sally had $2\frac{1}{3}$ jugs of lemonade before the party. After the party there were $1\frac{2}{3}$ jugs left. How much lemonade was used up during the party?



Method 1

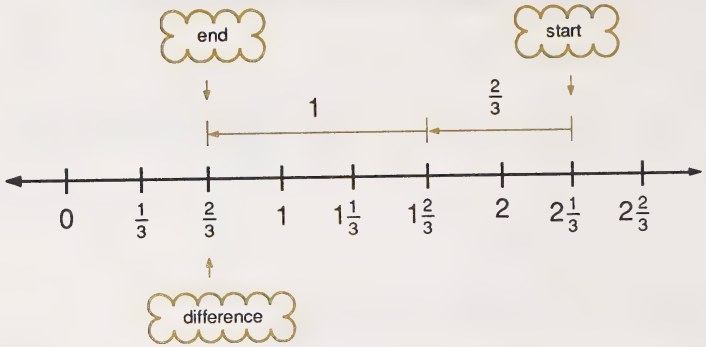
When adding vertically, work from right to left.

$$\begin{array}{rcl} 2\frac{1}{3} & = & 1\frac{4}{3} \\ -1\frac{2}{3} & = & -1\frac{2}{3} \\ \hline & & \frac{2}{3} \end{array}$$

First subtract fractions. Then subtract whole numbers.

After the party Sally found that $\frac{2}{3}$ of a jug of lemonade was used up during the party.

A number line may also be used to help you visualize the situation.



Method 2

When subtracting horizontally, work from left to right.

$$2\frac{1}{3} - 1\frac{2}{3}$$

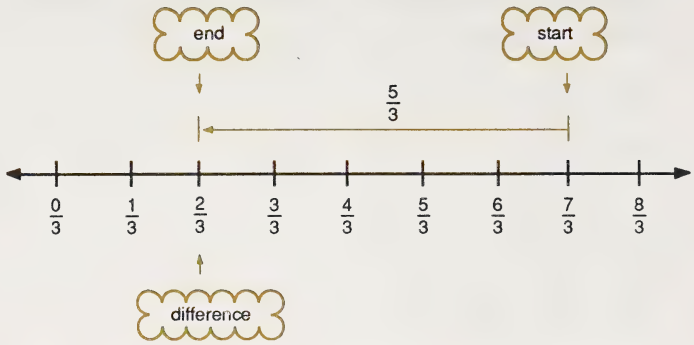
$$= \frac{7}{3} - \frac{5}{3}$$

$$= \frac{7 - 5}{3}$$

$$= \frac{2}{3}$$

Sally found that $\frac{2}{3}$ of a jug of lemonade was used up during the party.

A number line can be used to help you visualize the situation.



Practice Activities

Space for Your Work

1. Add each of the following. Express all final answers in simplest form.

   a. $1\frac{3}{5} + 2\frac{4}{5}$

   b. $2\frac{5}{6} + 1\frac{1}{6}$

   c. $3\frac{3}{4} + 2\frac{3}{4}$

   d. $5\frac{7}{8} + 2\frac{3}{8}$

   e. $1\frac{5}{12} + 5\frac{11}{12}$

2. Subtract each of the following. Express all final answers in simplest form.

   a. $4\frac{5}{8} - 1\frac{3}{8}$

   b. $2\frac{1}{4} - 1\frac{3}{4}$

   c. $3\frac{2}{5} - 1\frac{3}{5}$

   d. $4\frac{7}{12} - 2\frac{11}{12}$

   e. $2\frac{1}{10} - \frac{7}{10}$

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

1. Add each of the following. Express all final answers in simplest form.

   a. $\frac{3}{8} + \frac{3}{8}$

   b. $\frac{3}{7} + \frac{2}{7}$

   c. $\frac{4}{9} + \frac{7}{9}$

   d. $\frac{3}{10} + \frac{7}{10}$

   e. $3\frac{1}{5} + 1\frac{3}{5}$

   f. $4\frac{3}{8} + 5\frac{7}{8}$

   g. $1\frac{3}{5} + 2\frac{4}{5}$

   h. $2\frac{7}{8} + 1\frac{3}{8}$

2. Subtract each of the following. Express all final answers in simplest form.

Space for Your Work

a. $\frac{3}{4} - \frac{1}{4}$

b. $\frac{5}{8} - \frac{3}{8}$

c. $\frac{7}{9} - \frac{2}{9}$

d. $1\frac{3}{10} - \frac{7}{10}$

e. $3\frac{3}{5} - 1\frac{1}{5}$

f. $2\frac{7}{8} - 1\frac{5}{8}$

g. $2\frac{1}{4} - 1\frac{3}{4}$

h. $3\frac{2}{7} - 1\frac{5}{7}$

3. Jane listened to her radio for one hour or 60 minutes. She discovered that music was on for $\frac{35}{60}$ of the time, news used $\frac{5}{60}$ of the time, weather was given for $\frac{2}{60}$ of the time, sports highlights were reported for $\frac{3}{60}$ of the time, and commercials used up $\frac{15}{60}$ of the total time.

   a. What fraction of the total time was used for news, weather, and sports?

   b. How much more time was used for playing music than for advertising?

Space for Your Work

4. Melody had a jewellery collection that she wanted to share with her sisters. She decided to give $\frac{3}{10}$ of the collection to Judy and $\frac{5}{10}$ to June.

   a. How much of her jewellery collection did Melody give away?

   b. How much of the collection did she have left for herself?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

The following chart-like diagram is a **subtraction magic square**.

$\frac{2}{7}$	$\frac{1}{7}$	$\frac{4}{7}$
$\frac{3}{7}$	$\frac{5}{7}$	$\frac{7}{7}$
$\frac{6}{7}$	$\frac{9}{7}$	$\frac{8}{7}$

In a subtraction magic square, you follow these steps for each row, column, and diagonal of the square.

Step 1: Find the sum of the two end numbers.

Step 2: Subtract the middle number from the sum.

All the differences will be the same. This difference is called the **magic difference**.

Row 1: $\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$; $\frac{6}{7} - \frac{1}{7} = \frac{5}{7}$

Row 2: $\frac{3}{7} + \frac{7}{7} = \frac{10}{7}$; $\frac{10}{7} - \frac{5}{7} = \frac{5}{7}$

Row 3: $\frac{6}{7} + \frac{8}{7} = \frac{14}{7}$; $\frac{14}{7} - \frac{9}{7} = \frac{5}{7}$

Column 1: $\frac{2}{7} + \frac{6}{7} = \frac{8}{7}$; $\frac{8}{7} - \frac{3}{7} = \frac{5}{7}$

Column 2: $\frac{1}{7} + \frac{9}{7} = \frac{10}{7}$; $\frac{10}{7} - \frac{5}{7} = \frac{5}{7}$

Column 3: $\frac{4}{7} + \frac{8}{7} = \frac{12}{7}$; $\frac{12}{7} - \frac{7}{7} = \frac{5}{7}$

Diagonal starting at upper left

$\frac{2}{7} + \frac{8}{7} = \frac{10}{7}$; $\frac{10}{7} - \frac{5}{7} = \frac{5}{7}$

Diagonal starting at upper right

$\frac{4}{7} + \frac{6}{7} = \frac{10}{7}$; $\frac{10}{7} - \frac{5}{7} = \frac{5}{7}$

The magic difference is $\frac{5}{7}$.

Concluding Activities

Space for Your Work

Tell whether each of the following is a subtraction magic square.

1.

$\frac{6}{8}$	$\frac{10}{8}$	$\frac{8}{8}$
$\frac{4}{8}$	$\frac{6}{8}$	$\frac{8}{8}$
$\frac{2}{8}$	$\frac{2}{8}$	$\frac{4}{8}$

2.

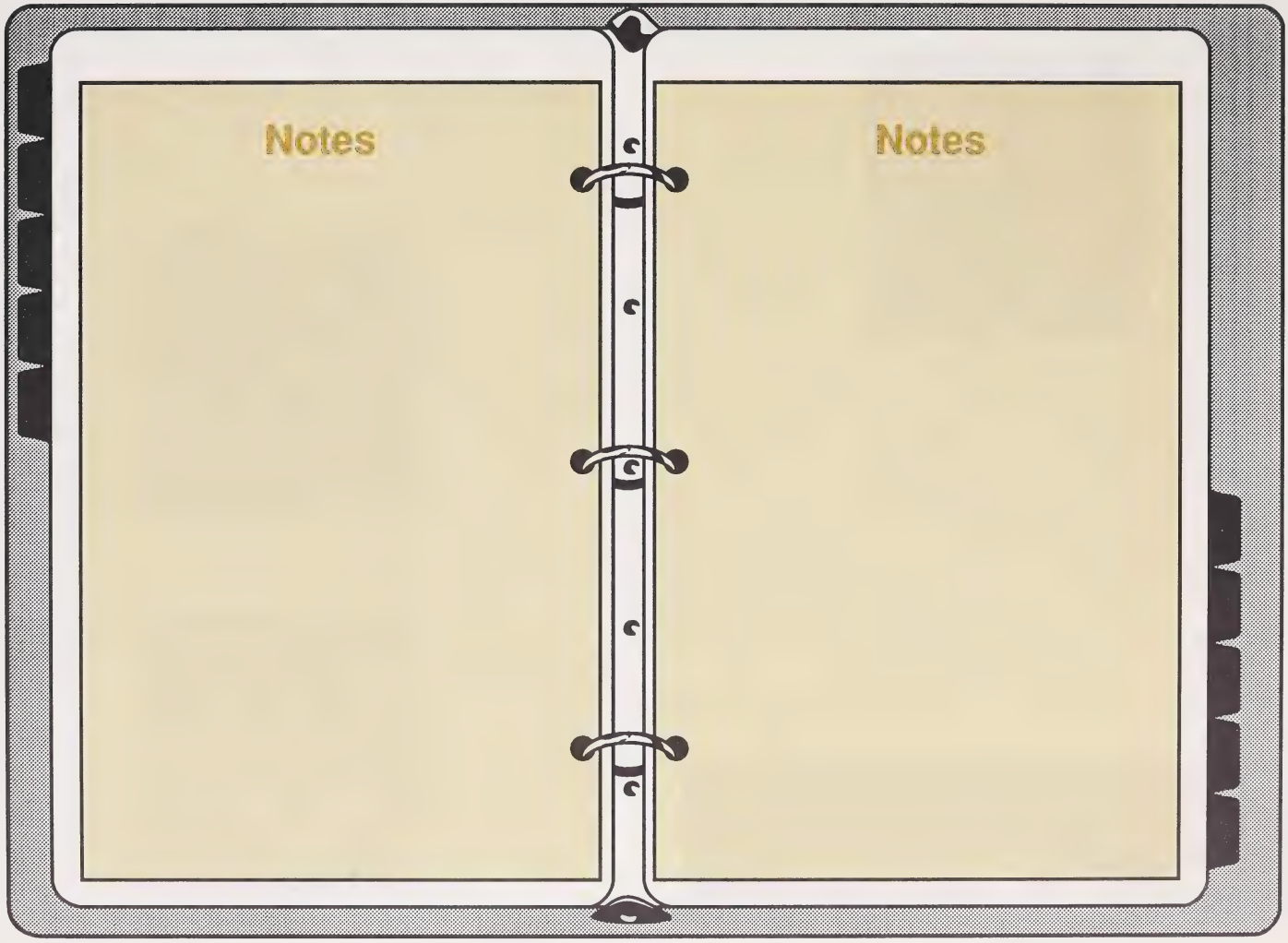
$\frac{6}{9}$	$\frac{3}{9}$	$\frac{12}{9}$
$\frac{9}{9}$	$\frac{15}{9}$	$\frac{21}{9}$
$\frac{18}{9}$	$\frac{27}{9}$	$\frac{24}{9}$

3.

Space for Your Work

$\frac{1}{6}$	$\frac{2}{6}$	$\frac{4}{6}$
$\frac{3}{6}$	$\frac{5}{6}$	$\frac{5}{6}$
$\frac{6}{6}$	$\frac{11}{6}$	$\frac{8}{6}$

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will learn these skills.

- estimating sums and differences of fractions and mixed numbers
- finding sums and differences using paper and pencil methods
- finding sums and differences using calculators

Working Together

In the last section you learned to add and subtract fractions and mixed numbers with like denominators. Now the real challenge will begin when you will add and subtract fractions and mixed numbers with unlike denominators.

Video Activity

Watch the video program *MATHWORKS: Adding and Subtracting Fractions and Mixed Numbers with Unlike Denominators.*

If you cannot watch this video, continue to read the notes which follow.

Example 1

The gas tank in Randy's car is $\frac{1}{4}$ full. To this he adds another $\frac{1}{2}$ tank of gas. How much gas does he have now?

Solution

Estimate by rounding to the nearest whole number.

$$\frac{1}{4} + \frac{1}{2} \doteq 0 + 1$$

$$\doteq 1$$

Calculate the exact answer.

$$\frac{1}{4} + \frac{1}{2}$$

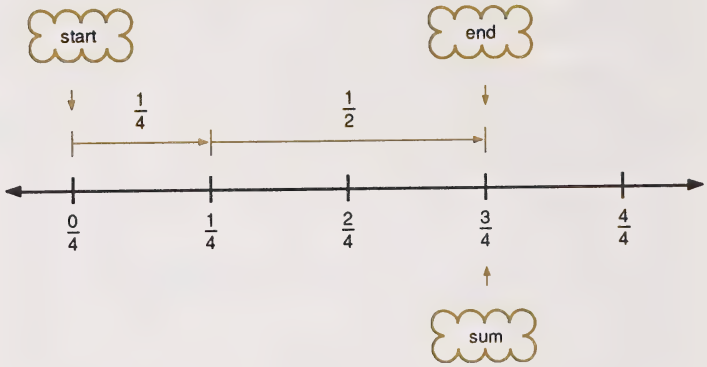
$$= \frac{1}{4} + \frac{2}{4} \quad \leftarrow \text{Change to equivalent fractions with like denominators.}$$

$$= \frac{1+2}{4} \quad \leftarrow \text{This step can be done mentally.}$$

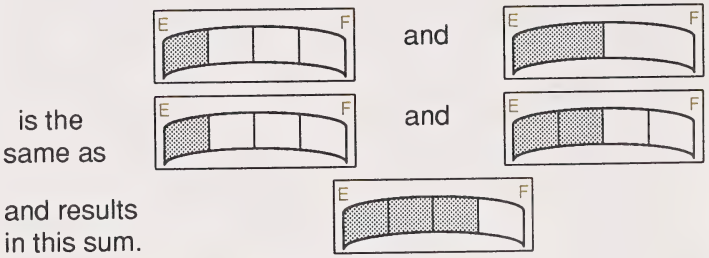
$$= \frac{3}{4}$$

Randy has $\frac{3}{4}$ of a tank of gas after adding another $\frac{1}{2}$ tank to the $\frac{1}{4}$ tank that he had originally.

A number line can be used to help you understand the situation.



A diagram can also be used to help you visualize the situation.



Example 2

The gas tank in Randy's car was $\frac{7}{8}$ full when he left home. When he got to his cottage the tank was $\frac{1}{2}$ full. How much gas did he use going to his cottage?

Solution

Estimate by rounding to the nearest whole number.

$$\frac{7}{8} - \frac{1}{2} \doteq 1 - 1$$

$$\doteq 0$$

Calculate the exact answer.

$$\frac{7}{8} - \frac{1}{2}$$

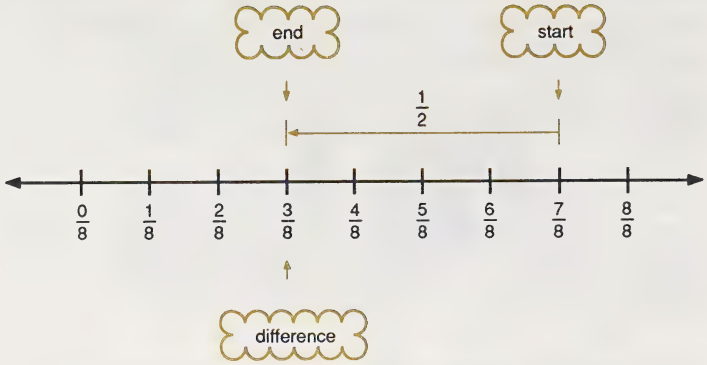
$$= \frac{7}{8} - \frac{4}{8}$$ ← Change to equivalent fractions with like denominators.

$$= \frac{7 - 4}{8}$$ ← This step can be done mentally.

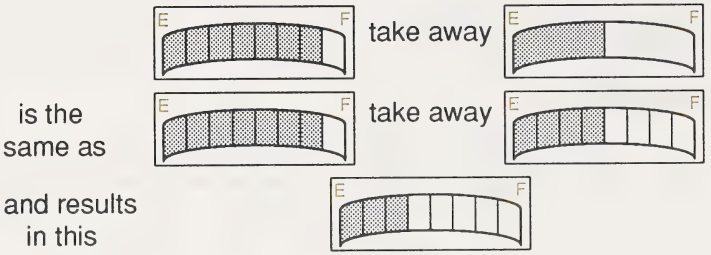
$$= \frac{3}{8}$$

Randy used $\frac{3}{8}$ of a tank of gas during his trip to the cottage.

A number line can be used to help you understand the situation.



A diagram can also be used to help you visualize the situation.



Example 3

Phoebe's St. Bernard ate $2\frac{1}{2}$ cans of dog food. Her collie ate $1\frac{1}{3}$ cans of dog food. How many cans of food did the two pets eat altogether?

Solution

Estimate by rounding to the nearest whole number.

$$2\tfrac{1}{2} + 1\tfrac{1}{3} \doteq 3 + 1$$

$$\doteq 4$$

Calculate the exact answer.

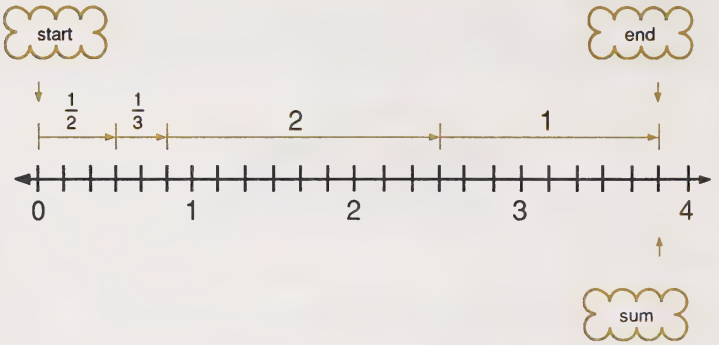
Method 1

$$\begin{array}{rr} 2\frac{1}{2} = & 2\frac{3}{6} \\ + \; 1\frac{1}{3} = & + 1\frac{2}{6} \\ \hline & 3\frac{5}{6} \end{array}$$

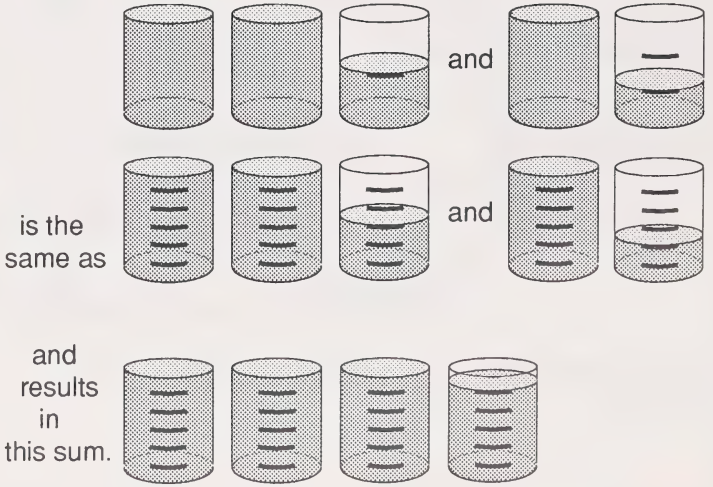
Add fractions. Then add whole numbers.

The two dogs ate $3\frac{5}{6}$ cans of dog food.

A number line can be used to help you understand the situation.



A diagram can also be used to help you visualize the situation.



Method 2

$$2\tfrac{1}{2} + 1\tfrac{1}{3}$$

$$= \tfrac{5}{2} + \tfrac{4}{3}$$

$$= \tfrac{15}{6} + \tfrac{8}{6}$$

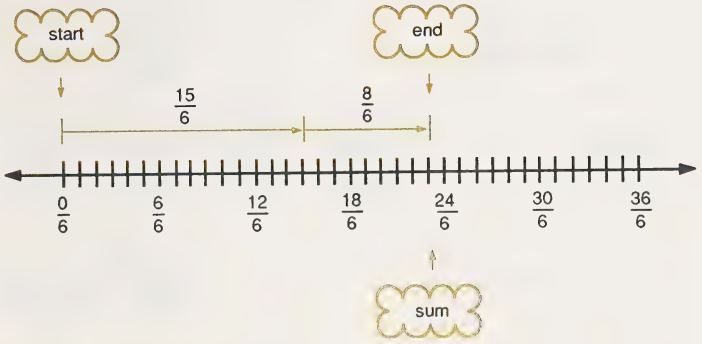
$$= \tfrac{23}{6}$$

$$= 3\tfrac{5}{6}$$

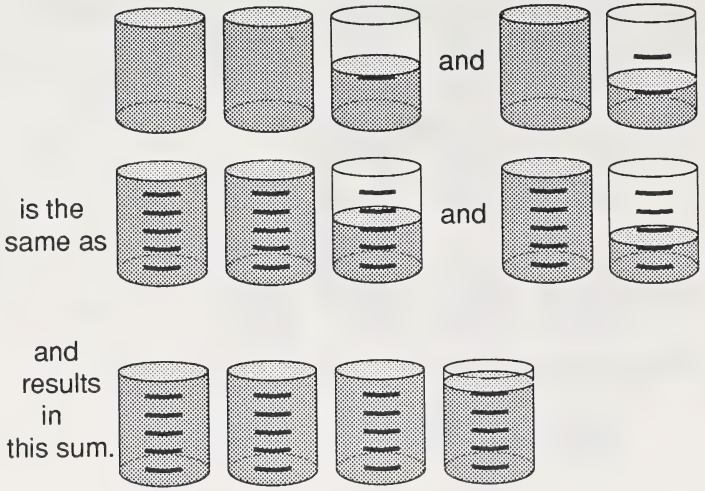
Change mixed numbers to improper fractions.

The two dogs ate $3\frac{5}{6}$ cans of dog food.

A number line can be used to help you understand the situation.



A diagram can also be used to help you visualize the situation.



Example 4

Marc drew $3\frac{3}{4}$ buckets of water from a well. He managed to spill $1\frac{1}{2}$ buckets of water. How many buckets of water did he have left?

Solution

Estimate by rounding to the nearest whole number.

$$3\tfrac{3}{4} - 1\tfrac{1}{2} \doteq 4 - 2$$

$$\doteq 2$$

Marc has about two buckets of water left.

Method 1

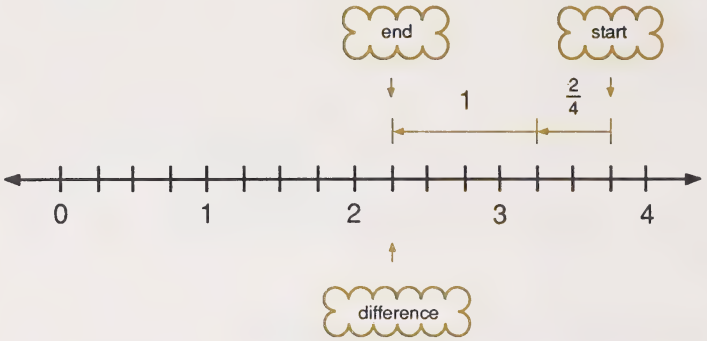
Calculate the exact answer.

$$\begin{array}{r} 3\frac{3}{4} = 3\frac{3}{4} \\ -1\frac{1}{2} = -1\frac{2}{4} \\ \hline 2\frac{1}{4} \end{array}$$

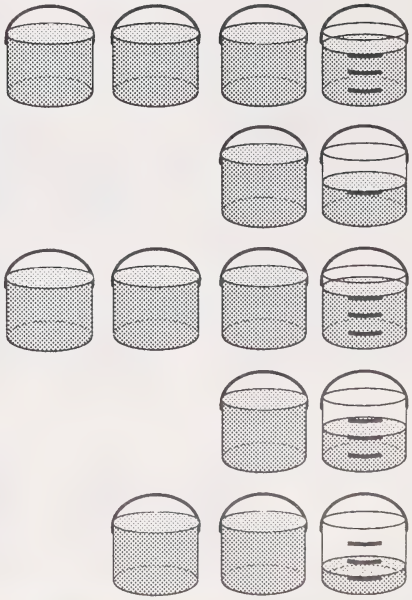
Subtract the fractions. Then subtract whole numbers.

Marc has $2\frac{1}{4}$ buckets of water left.

A number line can be used to help you understand the situation.



A diagram can also be used to help you visualize the situation.



Method 2

$$3\frac{3}{4} - 1\frac{1}{2}$$

$$= \frac{15}{4} - \frac{3}{2}$$

$$= \frac{15}{4} - \frac{6}{4}$$

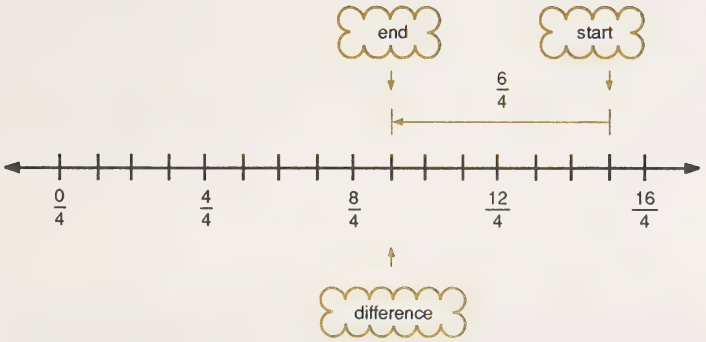
$$= \frac{9}{4}$$

$$= 2\frac{1}{4}$$

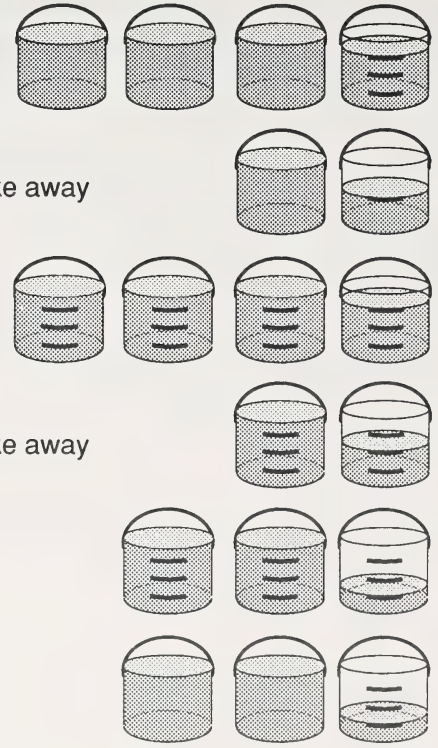
Change mixed numbers to improper fractions.

Marc has $2\frac{1}{4}$ buckets of water left.

A number line can be used to help you understand the situation.



A diagram can also be used to help you visualize the situation.



Introductory Activities

Space for Your Work

1. For each of the following, give an estimate first and then calculate the exact answer. Express all answers in simplest form.

   a. $\frac{1}{8} + \frac{1}{4}$

   b. $\frac{2}{3} + \frac{1}{6}$

   c. $2\frac{1}{3} + \frac{5}{12}$

   d. $3\frac{2}{5} + 4\frac{1}{2}$

   e. $2\frac{3}{8} + 1\frac{1}{4}$

2. For each of the following, give an estimate first and then calculate the exact answer. Express all answers in simplest form.

Space for Your Work

a. $\frac{4}{7} - \frac{1}{2}$

b. $\frac{7}{8} - \frac{1}{4}$

c. $\frac{1}{2} - \frac{1}{3}$

d. $3\frac{7}{8} - 1\frac{1}{2}$

e. $6\frac{7}{9} - 3\frac{2}{3}$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Regrouping

In the Introductory Activities, none of the examples required regrouping. Consider the following examples which involve **carrying** and **borrowing**.

Example 1

Minal has a brooch with three diamonds. One diamond is $1\frac{1}{2}$ carats. The other two diamonds are $\frac{3}{4}$ carats each. What is the total weight of the diamonds?

Solution

Estimate by rounding to the nearest whole number.

$$1\tfrac{1}{2} + \tfrac{3}{4} + \tfrac{3}{4} \doteq 2 + 1 + 1$$

$$\doteq 4$$

The total weight of the diamonds is about four carats.

Method 1

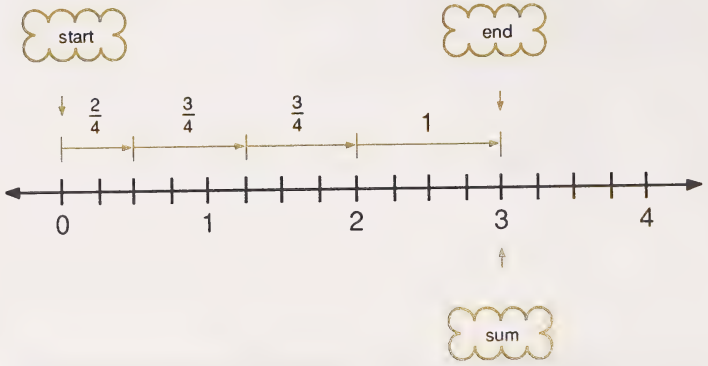
Calculate the exact answer.

$$\begin{array}{r} 1\frac{1}{2} = 1\frac{2}{4} \\ \frac{3}{4} = \frac{3}{4} \\ +\ \frac{3}{4} = +\ \frac{3}{4} \\ \hline 1\frac{8}{4} = 1 + 2 \\ = 3 \end{array}$$

Add fractions first. Then add whole numbers.

The total weight of the diamonds is three carats.

A number line can help you understand the situation.



Method 2

Calculate the exact answer.

$$1\frac{1}{2} + \frac{3}{4} + \frac{3}{4}$$

$$= \frac{3}{2} + \frac{3}{4} + \frac{3}{4}$$

Change mixed numbers to improper fractions.

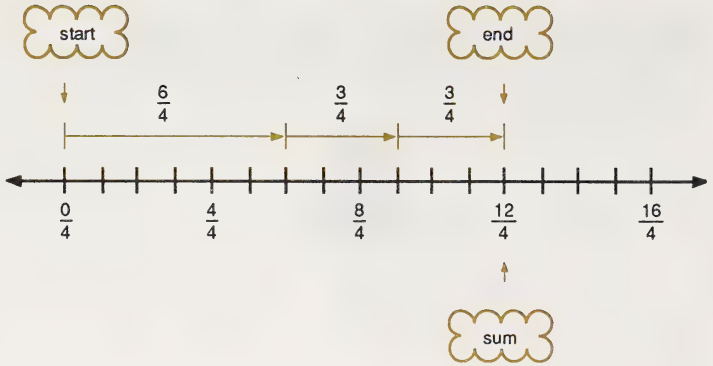
$$= \frac{6}{4} + \frac{3}{4} + \frac{3}{4}$$

$$= \frac{12}{4}$$

$$= 3$$

The total weight of the diamonds is three carats.

A number line can help you understand the situation.



Example 2

Jason had four patio decks to paint. If he painted $1\frac{1}{2}$ decks on Saturday, how many must he still paint to complete the job?

Solution

Estimate by rounding to the nearest whole number.

$$4 - 1\frac{1}{2} \doteq 4 - 2$$

$$\doteq 2$$

Jason has about two patio decks left to paint.

Method 1

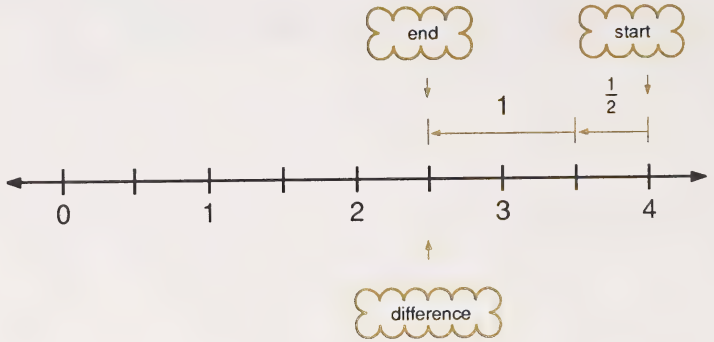
Calculate the exact answer.

$$\begin{array}{rr} 4 = & 3\frac{2}{2} \\ -1\frac{1}{2} = & -1\frac{1}{2} \\ \hline & 2\frac{1}{2} \end{array}$$

Subtract fractions. Then subtract whole numbers.

Jason has $2\frac{1}{2}$ patio decks left to paint.

A number line will help you understand the situation.



Method 2

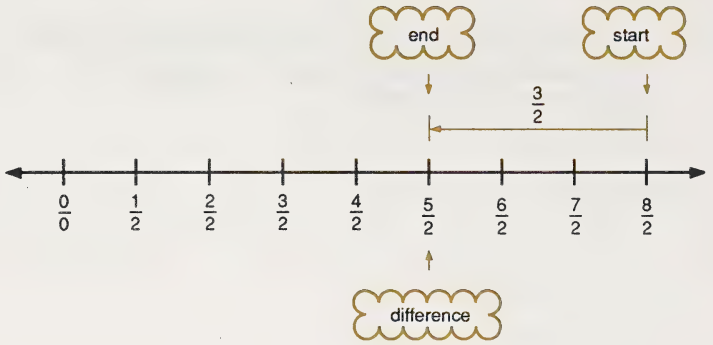
Calculate the exact answer.

$$4 - 1\frac{1}{2}$$
$$= 4 - \frac{3}{2}$$
$$= \frac{8}{2} - \frac{3}{2}$$
$$= \frac{5}{2}$$
$$= 2\frac{1}{2}$$

Change mixed numbers to improper fractions.

Jason has $2\frac{1}{2}$ patio decks left to paint.

A number line will help you understand the situation.



Practice Activities

Space for Your Work

1. For each of the following, give an estimate first and then calculate the exact answer. Express all answers in simplest form.

   a. $\frac{1}{2} + \frac{1}{4}$

   b. $\frac{5}{8} + \frac{3}{5}$

   c. $\frac{5}{6} + \frac{4}{9}$

   d. $2\frac{4}{5} + 3\frac{7}{10}$

   e. $4\frac{5}{6} + 3\frac{3}{4}$

   f. $2\frac{1}{2} + 5\frac{2}{3}$

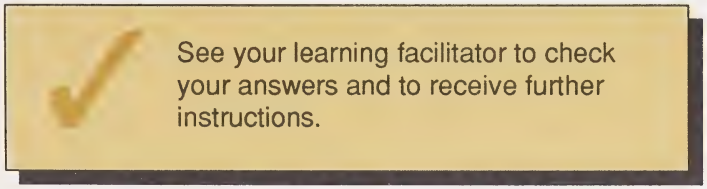
2. For each of the following, give an estimate first and then calculate the exact answer. Express all answers in simplest form.

   a. $3\frac{1}{10} - 1\frac{3}{5}$

   b. $4\frac{1}{6} - 1\frac{2}{3}$

   c. $5\frac{5}{6} - \frac{8}{9}$

   d. $1\frac{1}{4} - \frac{1}{2}$



See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Do Question 1 or Questions 2 to 5.

Computer Alternative

1. a. Do adding and subtracting on the disk *Conquering Fractions (+,-)* by MECC. Press [V] to view any of the problems in picture form.

   b. Do Lessons 7, 8, 9, 10, and 11 of the disk *Fractions D* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

In this program you must type [R] to work step by step on the screen. After pressing [R] , use [→] to move from step to step.

If you wish to do the computation mentally and you simply want to type in the answer, do not press [R] . Press the [→] key to move the cursor to the numerator or to the denominator. Remember to hold down the SHIFT key and press [?] to receive help.

Space for Your Work

Space for Your Work

2. Add each of the following. Express all final answers in simplest form.

a. $\frac{4}{10} + \frac{5}{10}$

b. $\frac{1}{3} + \frac{3}{10}$

c. $\frac{3}{8} + \frac{3}{4}$

d. $1\frac{4}{5} + 3\frac{2}{5}$

e. $2\frac{5}{6} + 5\frac{1}{2}$

3. Subtract each of the following. Express all final answers in simplest form.

a. $\frac{5}{6} - \frac{3}{6}$

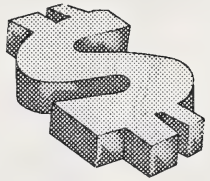
b. $\frac{2}{3} - \frac{3}{5}$

c. $1\frac{3}{4} - 1\frac{1}{4}$

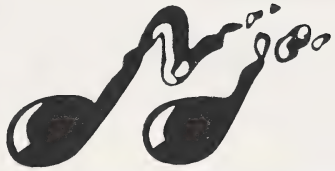
d. $5\frac{3}{8} - 2\frac{1}{2}$

e. $10\frac{2}{5} - 8\frac{2}{3}$

4. The Canadian Cancer Society collected $10\frac{1}{2}$ million dollars in a fund-raising drive. The Heart Foundation collected $8\frac{1}{3}$ million dollars. In all, how much money was collected by the two agencies?



5. The Brass Quartet received a $3\frac{1}{2}$ minute ovation. The String Trio received a $4\frac{1}{4}$ minute ovation. How much longer was the String Trio's ovation?



Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Accuracy with Fractions and Decimal Numbers

If you want to add or subtract fractions that will produce terminating decimals, you may use either the fraction or decimal number form.

Example: Find the sum of $\frac{1}{2} + \frac{1}{8}$.

Solution

$\frac{1}{2} + \frac{1}{8}$	$\frac{1}{2} = 0.5$
$= \frac{4}{8} + \frac{1}{8}$	$\frac{1}{8} = 0.125$
$= \frac{5}{8}$	$0.5 \times 0.125 = 0.625$

$$\frac{5}{8} = 0.625$$

Both $\frac{5}{8}$ and 0.625 are exact sums.

If you need to add or subtract fractions, and one or more of the fractions produces a repeating decimal, you must use the fraction form for an accurate answer. The decimal number forms would produce an approximate answer.

Example: Find the sum of $\frac{1}{3} + \frac{1}{2}$.

Solution

$\frac{1}{3} + \frac{1}{2}$	$\frac{1}{3} \doteq 0.3$
$= \frac{2}{6} + \frac{3}{6}$	$\frac{1}{2} = 0.5$
$= \frac{5}{6}$	$0.3 + 0.5 \doteq 0.8$

$$\frac{5}{6} \neq 0.8$$

$\frac{5}{6}$ is an exact sum.

0.8 is an approximate sum.

Some scientific calculators allow you to add and subtract fractions. These calculators have the $a\frac{b}{c}$ key. Here is how these calculators work. (If the steps outlined in the examples do not work on your calculator, refer to the Owner's Manual for your calculator.)

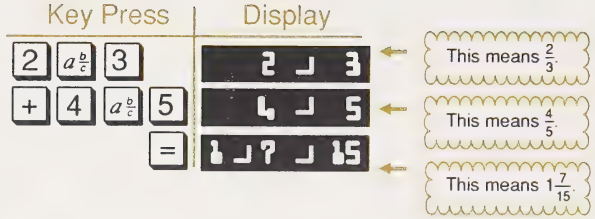
Example 1: What is $\frac{1}{4} + \frac{1}{3}$?

So, $\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$.

Example 2: What is $\frac{7}{9} - \frac{2}{3}$?

So, $\frac{7}{9} - \frac{2}{3} = \frac{1}{9}$.

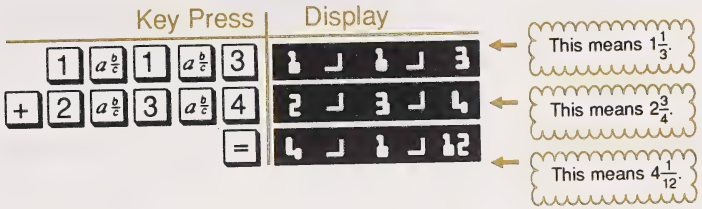
Example 3: What is $\frac{2}{3} + \frac{4}{5}$?



So, $\frac{2}{3} + \frac{4}{5} = 1\frac{7}{15}$.

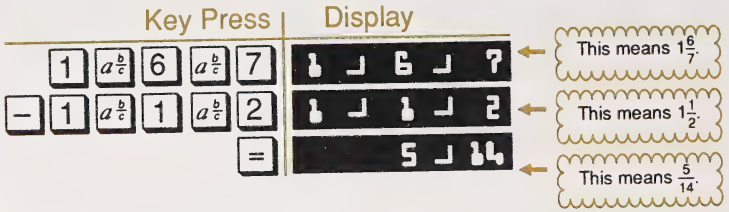
You can add and subtract mixed numbers on a calculator, too.

Example 1: What is $1\frac{1}{3} + 2\frac{3}{4}$?



So, $1\frac{1}{3} + 2\frac{3}{4} = 4\frac{1}{12}$.

Example 2: What is $1\frac{6}{7} - 1\frac{1}{2}$?



So, $1\frac{6}{7} - 1\frac{1}{2} = \frac{5}{14}$.

Concluding Activities

Space for Your Work

1. Evaluate the following with a calculator using the $a\frac{b}{c}$ key.

   a. $\frac{1}{8} + \frac{1}{4}$

   b. $\frac{2}{3} - \frac{1}{6}$

   c. $1\frac{1}{2} + \frac{3}{10}$

   d. $1\frac{3}{7} - \frac{3}{4}$

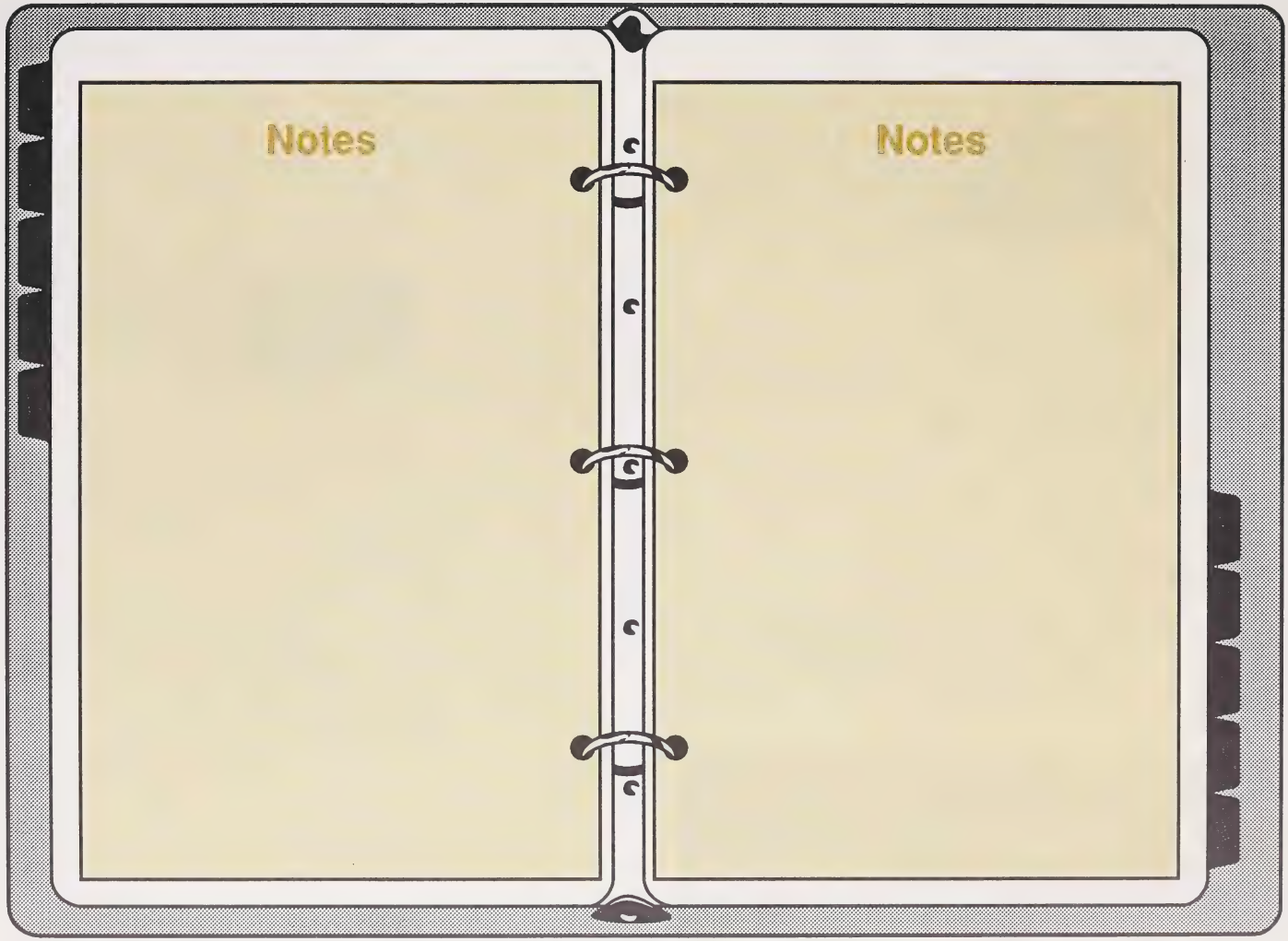
   e. $1\frac{1}{6} + 2\frac{2}{3}$

   f. $3\frac{3}{4} - 2\frac{7}{8}$

Computer Alternative

2. For a fun game of estimation play *Fraction Hunt* on Disk C of MAC 6 (Houghton Mifflin).

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will review strategies used to mentally add and subtract whole numbers and extend these strategies to include mentally adding and subtracting fractions and mixed numbers.

Working Together

There are several ways to obtain exact sums and differences.

- use paper and pencil computation
- use a calculator
- use mental computation

It is important that you are skillful in all three ways. You have practised finding exact sums and differences using paper and pencil methods and using a calculator. Now you will practise computing exact sums and differences mentally.

In previous schooling you have learned to add and subtract mentally with whole numbers. The first part of this section will review the strategies for adding and subtracting whole numbers mentally.

Video Activity

Watch the videos *Solve It: Using Mental Computation for Addition* (AIT) and *Solve It: Using Mental Computation for Subtraction* (AIT). If you cannot watch the videos, read the notes which follow.

The Plus-Minus Method for Addition

In order to avoid regrouping, you can add or subtract a number in one addend as long as you do the opposite to the other addend.

Example 1: What is 98 + 33?

Think

$$98 + 33 = 100 + 31$$
$$= 131$$

Example 2: What is 32 + 59?

Think

$$32 + 59 = 30 + 61$$
$$= 91$$

The Attention Method for Subtraction

In order to avoid regrouping, you may add or take away a number in the subtrahend as long as you do the same to the minuend.

Example 1: What is 180 − 71?

Think

$$180 - 71 = 179 - 70$$
$$= 109$$

Example 2: What is 180 − 69?

Think

$$180 - 69 = 181 - 70$$
$$= 111$$

Introductory Activities

Space for Your Work

1. Use the plus-minus method to mentally find the exact sums.

   a. 38 + 19

   b. 99 + 48

   c. 38 + 66

   d. 46 + 125

   e. 122 + 218

   f. 828 + 142

2. Use the attention method to mentally find the exact differences.

   a. 56 – 19

   b. 84 – 36

   c. 91 – 53

   d. 146 – 38

   e. 370 – 102

   f. 765 – 215

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Mental Computation with Mixed Numbers

Now that you have used mental strategies for addition and subtraction of whole numbers, apply these strategies to include addition and subtraction of mixed numbers.

The Plus-Minus Method for Addition

Example 1: What is $3\frac{7}{8} + 2\frac{5}{8}$?

Solution

Think $3\frac{7}{8} + \frac{1}{8} = 4$

$2\frac{5}{8} - \frac{1}{8} = +2\frac{4}{8}$

$6\frac{4}{8}$ or $6\frac{1}{2}$

To avoid regrouping $\frac{1}{8}$ was added to $3\frac{7}{8}$, and $\frac{1}{8}$ was subtracted from $2\frac{5}{8}$ to keep the equality.

Example 2: What is $5\frac{3}{4} + 4\frac{1}{2}$?

Solution

Think $5\frac{3}{4} + \frac{1}{4} = 6$

$4\frac{1}{2} - \frac{1}{4} = +4\frac{1}{4}$

$10\frac{1}{4}$

In order to avoid regrouping $\frac{1}{4}$ was added to $5\frac{3}{4}$, and $\frac{1}{4}$ was subtracted from $4\frac{1}{2}$ to maintain equality.

You may have to use paper and pencil methods to find equivalent fractions with common denominators. You would write

$$4\frac{1}{2} - \frac{1}{4} = 4\frac{2}{4} - \frac{1}{4}$$

$$= 4\frac{1}{4}.$$

The rest can be done in your head.

The Attention Method for Subtraction

Example 1: What is $5\frac{3}{7} - 2\frac{5}{7}$?

Solution

Think $5\frac{3}{7} + \frac{2}{7} = 5\frac{5}{7}$

$2\frac{5}{7} + \frac{2}{7} = -3$

$2\frac{5}{7}$

In order to avoid borrowing, $\frac{2}{7}$ was added to $2\frac{5}{7}$. In order to maintain equality, $\frac{2}{7}$ was also added to $5\frac{3}{7}$.

Example 2: What is $6\frac{1}{4} - 3\frac{1}{2}$?

Solution

Think $6\frac{1}{4} + \frac{1}{2} = 6\frac{3}{4}$

$3\frac{1}{2} + \frac{1}{2} = -4$

$2\frac{3}{4}$

To avoid borrowing, $\frac{1}{2}$ was added to $3\frac{1}{2}$. To maintain equality, $\frac{1}{2}$ was also added to $6\frac{1}{4}$.

You may need to use paper and pencil methods to find equivalent fractions. You would write this.

$$6\frac{1}{4} + \frac{1}{2} = 6\frac{1}{4} + \frac{2}{4}$$

Then you would do the rest mentally.

Practice Activities

Space for Your Work

1. Use the plus-minus method to mentally find the sum for each of the following.

   a. $6\frac{3}{8} + 5\frac{7}{8}$

   b. $7\frac{4}{7} + 2\frac{5}{7}$

   c. $7\frac{1}{2} + 1\frac{3}{4}$

   d. $2\frac{5}{9} + 3\frac{2}{3}$

2. Use the attention method to mentally find the difference for each of the following.

   a. $9\frac{2}{5} - 7\frac{4}{5}$

   b. $6\frac{1}{8} - 3\frac{5}{8}$

   c. $8\frac{1}{10} - 6\frac{3}{5}$

   d. $5\frac{1}{2} - 2\frac{3}{4}$

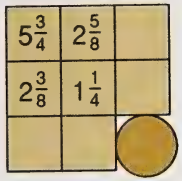
See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Fill the spaces by adding across and subtracting down. The result for the circle must be the same for the addition and the subtraction.

1.



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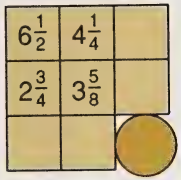

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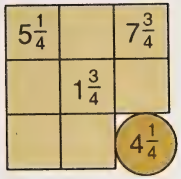
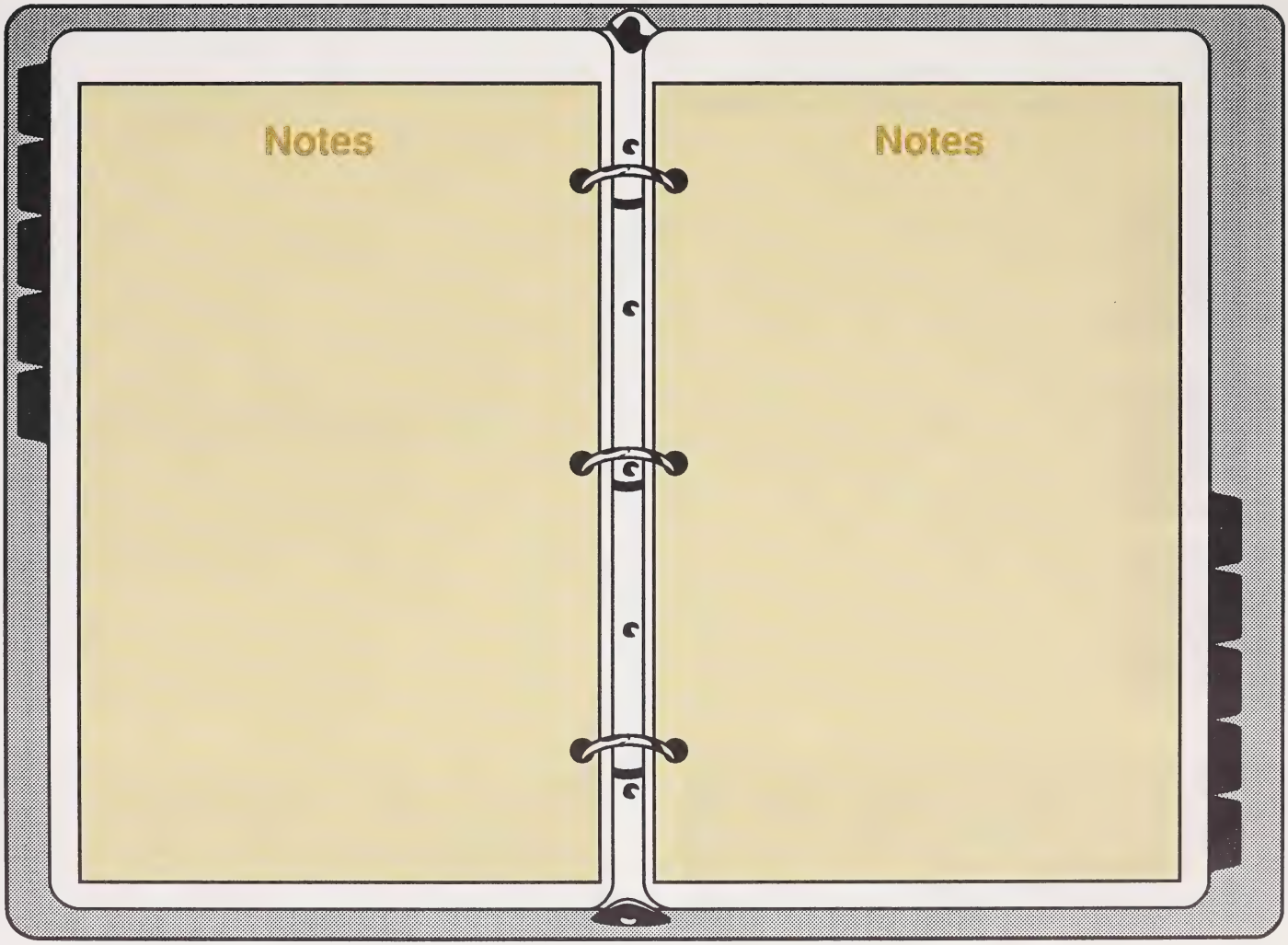


See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will learn these skills.

- estimating products of fractions and mixed numbers
- computing products of fractions and mixed numbers using paper and pencil methods
- computing products of fractions and mixed numbers using a calculator

Working Together

Multiplying is repeated addition. Keep this in mind as you do this section.

Examples

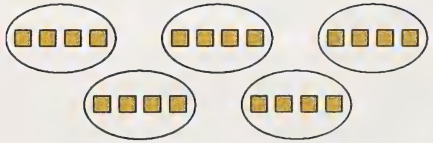
- 1×4 means one group of 4.

So, $1 \times 4 = 4$.

- 3×4 means three groups of 4.

So, $3 \times 4 = 4 + 4 + 4$ or 12.

- 5×4 means five groups of 4.



So, $5 \times 4 = 4 + 4 + 4 + 4 + 4$ or 20.

Introductory Activities

Space for Your Work

1. Find the product for each of the following.
   a. 4×6
   b. 3×6
   c. 2×6
   d. 1×6

2. Use the results from Question 1 to make the following prediction. Will $\frac{1}{3} \times 6$ be greater than 6 or less than 6?

3. a. If a number is multiplied by a number greater than 1, will the product be greater than itself, equal to itself, or less than itself?

   b. If a number is multiplied by 1, will the product be greater than itself, equal to itself, or less than itself?

   c. If a number is multiplied by a number less than 1, will the product be greater than itself, equal to itself, or less than itself?

See your learning facilitator to check your answers and to receive further instructions.

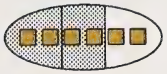
Working Together

Mixed Numbers

$\frac{1}{3} \times 6$ means one of three equal parts of 6.

So, $\frac{1}{3} \times 6 = 2$.

$\frac{2}{3} \times 6$ means two of three equal parts of 6.



So, $\frac{2}{3} \times 6 = 2 + 2$ or 4.

$2\frac{1}{2} \times 6$ means two groups of six and one of two equal parts of 6.

So, $2\frac{1}{2} \times 6 = 6 + 6 + 3$ or 15.

Video Activity

View the video *Solve It: Multiplication with Fractions and Mixed Numbers* (AIT). Then read the notes which follow. If you cannot view the video, go on and read the notes which come next.

In this example you are multiplying using a fraction.

Example

Rocco used $\frac{1}{3}$ of a can of paint to refinish each chair. If he refinished six chairs in all, how many cans of paint did he use?

Solution

Estimate the value of $6 \times \frac{1}{3}$. Since $6 \times 1 = 6$, and $\frac{1}{3}$ is less than 1, the product is less than 6.

Calculate the exact product.

$$6 \times \frac{1}{3}$$

$$= \frac{6}{1} \times \frac{1}{3}$$

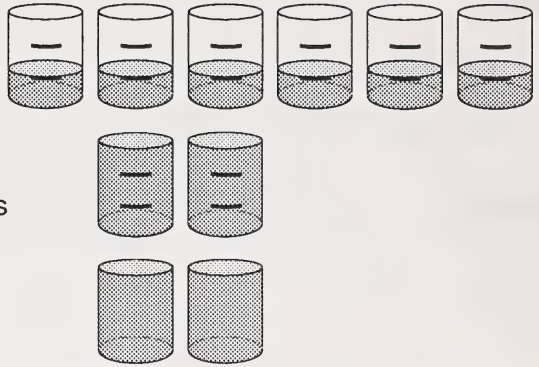
$$= \frac{6 \times 1}{1 \times 3}$$

$$= \frac{6}{3}$$

$$= 2$$

Multiply the numerators. Then multiply the denominators.

A diagram can be used to help you visualize the situation.



You should remember that multiplication and addition are related.

In the next example you are multiplying with a mixed number.

Example

Carlo reads one page of the newspaper in $3\frac{1}{2}$ minutes. How long would it take him to read five pages?

Solution

Estimate the value of $5 \times 3\frac{1}{2}$.

$5 \times 3 = 15$ and $5 \times 4 = 20$

So, the product is between 15 and 20.

Carlo would take between 15 and 20 minutes to read five pages of the newspaper.

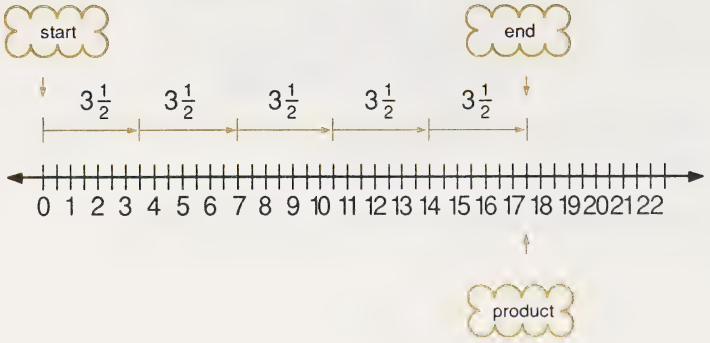
Calculate the exact product.

$$\begin{aligned} & 5 \times 3\tfrac{1}{2} \\ = & \tfrac{5}{1} \times \tfrac{7}{2} \\ = & \tfrac{5 \times 7}{1 \times 2} \\ = & \tfrac{35}{2} \\ = & 17\tfrac{1}{2} \end{aligned}$$

Multiply the numerators. Then multiply the denominators.

Carlo would take $17\frac{1}{2}$ minutes to read five pages of the newspaper.

A number line can be used to help you visualize the situation.



So, $3\frac{1}{2} \times 5 = 17\frac{1}{2}$.

In the example which follows you are multiplying with two fractions.

Example

Mr. Sosnowski rototilled $\frac{3}{5}$ of his garden. He then planted $\frac{1}{2}$ of the rototilled part of the garden. What fraction of the entire garden did he plant?

Solution

Estimate a product for $\frac{1}{2} \times \frac{3}{5}$.

$\frac{1}{2} \times \frac{3}{5}$ means one of two equal parts of $\frac{3}{5}$.

Since $\frac{3}{5}$ and $\frac{1}{2}$ are both less than one, the product will be less than $\frac{3}{5}$.

Mr. Sosnowski planted less than $\frac{3}{5}$ of his garden.

Calculate the exact product.

$$\frac{1}{2} \times \frac{3}{5}$$

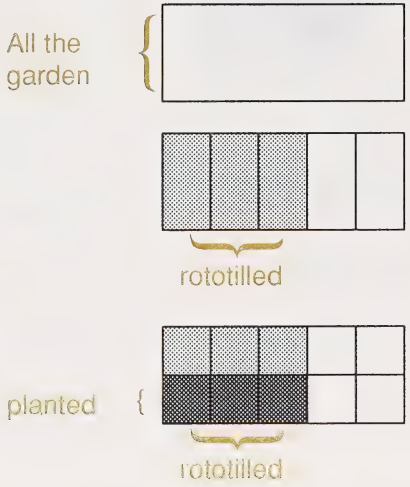
$$= \frac{1 \times 3}{2 \times 5}$$

$$= \frac{3}{10}$$

Multiply the numerators. Then multiply the denominators.

Mr. Sosnowski planted $\frac{3}{10}$ of his entire garden.

A diagram can be used to help you visualize the situation.



In the next example you are multiplying a mixed number with a fraction.

Example

Malanka knits $2\frac{3}{5}$ afghans in one month. How many afghans does she knit in $\frac{1}{2}$ of a month?

Solution

Estimate a product for $\frac{1}{2} \times 2\frac{3}{5}$. Since $1 \times 2\frac{3}{5} = 2\frac{3}{5}$ and $\frac{1}{2}$ is less than 1, the product is less than $2\frac{3}{5}$.

$$\frac{1}{2} \times 2\frac{3}{5} \doteq \frac{1}{2} \times 3$$

$$\doteq 1\frac{1}{2}$$

Malanka will knit about $1\frac{1}{2}$ afghans in $\frac{1}{2}$ of a month.

Calculate the exact product.

$$\frac{1}{2} \times 2\frac{3}{5}$$

$$= \frac{1}{2} \times \frac{13}{5}$$

$$= \frac{1 \times 13}{2 \times 5}$$

$$= \frac{13}{10}$$

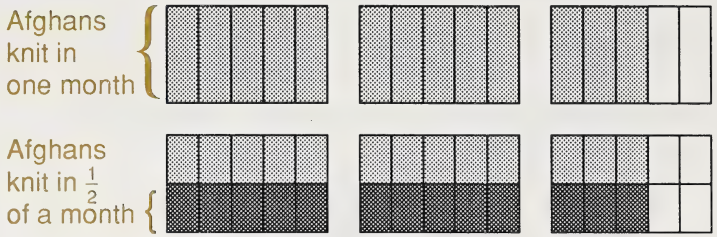
$$= 1\frac{3}{10}$$

Change $2\frac{3}{5}$ to an improper fraction.

Multiply the numerators. Then multiply the denominators.

Malanka knits $1\frac{3}{10}$ afghans in $\frac{1}{2}$ of a month.

A diagram can help you visualize the situation.



Now you will study an example where two mixed numbers are multiplied.

Example

Ruth can weld $2\frac{1}{2}$ kits in one hour. How many kits can she weld in $1\frac{1}{4}$ hours?

Solution

Estimate the product for $1\frac{1}{4} \times 2\frac{1}{2}$.

$$1\tfrac{1}{4} \times 2\tfrac{1}{2} \doteq 1 \times 3$$

$$\doteq 3$$

Ruth can weld about three kits in $1\frac{1}{4}$ hours.

Calculate the exact product.

$$1\tfrac{1}{4} \times 2\tfrac{1}{2}$$

$$= \tfrac{5}{4} \times \tfrac{5}{2}$$

$$= \tfrac{5 \times 5}{4 \times 2}$$

$$= \tfrac{25}{8}$$

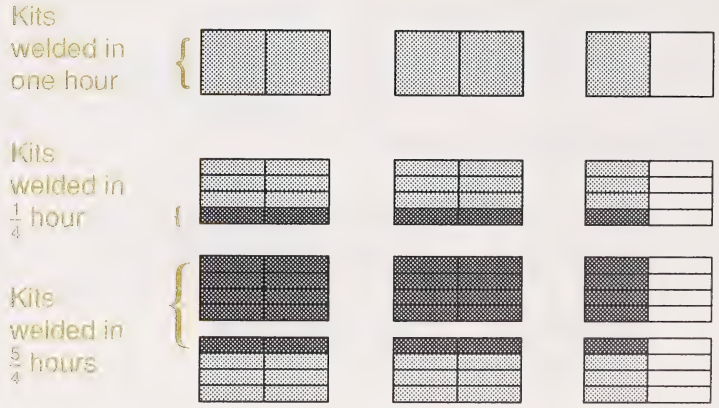
$$= 3\tfrac{1}{8}$$

Change $1\frac{1}{4}$ and $2\frac{1}{2}$ to improper fractions.

Multiply the numerators. Then multiply the denominators.

Ruth can weld $3\frac{1}{8}$ kits in $1\frac{1}{4}$ hours.

A diagram can be used to help you visualize the situation.



Practice Activities

Space for Your Work

1. Find the product for each of the following. Express all final answers in simplest form.

   a. $3 \times \frac{1}{2}$

   b. $5 \times \frac{2}{3}$

   c. $4 \times \frac{1}{5}$

   d. $20 \times \frac{3}{10}$

2. Find the product for each of the following. Express all final answers in simplest form.

   a. $3 \times 1\frac{1}{4}$

   b. $2 \times 3\frac{1}{6}$

   c. $5 \times 1\frac{2}{3}$

   d. $3 \times \frac{5}{8}$

3. Circle the letter of the question below which will produce a product larger than either of the two factors.

   a. $\frac{1}{5} \times \frac{2}{3}$

   b. $\frac{3}{8} \times 2\frac{1}{4}$

   c. $1\frac{1}{2} \times 3\frac{3}{5}$

4. Circle the letter of the question below which will produce a product smaller than either of the two factors.

   a. $\frac{1}{5} \times \frac{2}{3}$

   b. $\frac{3}{8} \times 2\frac{1}{4}$

   c. $1\frac{1}{2} \times 3\frac{3}{5}$

5. Find the product for each of the following. Express all final answers in simplest form.

   a. $\frac{1}{4} \times \frac{3}{5}$

   b. $\frac{1}{2} \times \frac{1}{6}$

   c. $\frac{1}{5} \times \frac{2}{3}$

   d. $\frac{3}{8} \times \frac{5}{8}$

Space for Your Work

6. Find the product for each of the following. Express all final answers in simplest form.

   a. $\frac{2}{3} \times 1\frac{1}{5}$

   b. $\frac{1}{3} \times 2\frac{1}{4}$

   c. $\frac{3}{8} \times 1\frac{1}{2}$

   d. $\frac{1}{4} \times 2\frac{4}{5}$

7. Find the product for each of the following. Express all final answers in simplest form.

   a. $2\frac{1}{2} \times 1\frac{5}{8}$

   b. $3\frac{1}{2} \times 2\frac{4}{5}$

   c. $4\frac{2}{3} \times 1\frac{1}{6}$

   d. $3\frac{1}{4} \times 2\frac{1}{2}$

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

Do Questions 1 or 2 and then do Question 3.

Computer Alternative

1. a. Do the multiplying program of the disk *Conquering Fractions* (MECC). Press [v] to view any of the problems in picture form. Do not press [s] at this time.

   b. Do Lessons 12, 13, and 14 of the disk *Fractions* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA). Read the instructions in the folder before using the program. If you need help, remember to hold down the SHIFT key and press the [?] key.

Space for Your Work

2. Find the product for each of the following. Express all final answers in simplest form.

a. $2 \times \frac{1}{3}$

b. $6 \times \frac{2}{5}$

c. $5 \times \frac{3}{4}$

d. $5 \times 1\frac{1}{4}$

e. $2 \times 1\frac{2}{3}$

f. $3 \times 2\frac{1}{2}$

3. Find the product for each of the following. Express all final answers in simplest form.

a. $\frac{1}{3} \times \frac{1}{2}$

b. $\frac{3}{4} \times \frac{3}{5}$

c. $\frac{4}{5} \times \frac{7}{6}$

d. $\frac{2}{7} \times \frac{1}{3}$

e. $\frac{1}{3} \times 2\frac{3}{4}$

f. $\frac{2}{3} \times 1\frac{3}{5}$

g. $\frac{4}{7} \times 1\frac{1}{6}$

h. $\frac{1}{3} \times 2\frac{1}{2}$

i. $1\frac{2}{3} \times 2\frac{1}{2}$

j. $1\frac{4}{5} \times 1\frac{1}{2}$

k. $3\frac{1}{2} \times 2\frac{7}{8}$

l. $2\frac{1}{4} \times 3\frac{3}{10}$

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Accuracy with Fractions and Decimal Numbers

If you want to multiply fractions that will produce terminating decimal numbers, you can use either the fraction or the decimal number form.

Example: Find the product of $\frac{1}{4}$ and $\frac{5}{8}$.

Solution

$\frac{1}{4} \times \frac{5}{8}$	$\frac{1}{4} = 0.25$
$= \frac{1 \times 5}{4 \times 8}$	$\frac{5}{8} = 0.625$
$= \frac{5}{32}$	$0.25 \times 0.625 = 0.156\ 25$

$0.156\ 25$

Both $\frac{5}{32}$ and $0.156\ 25$ are exact products.

If you need to multiply fractions, and one or more of the fractions produce a repeating decimal number, you must use the fraction form for an accurate answer. The decimal number forms would produce an approximate answer.

Example: Find the product of $\frac{2}{3}$ and $\frac{1}{2}$.

Solution

$\frac{2}{3} \times \frac{1}{2}$	$\frac{2}{3} \doteq 0.7$
$= \frac{2 \times 1}{3 \times 2}$	$\frac{1}{2} = 0.5$
$= \frac{2}{6}$	$0.7 \times 0.5 \doteq 0.35$
$= \frac{1}{3}$	

$\frac{1}{3} \neq 0.35$

$\frac{1}{3}$ is an exact product.

0.35 is an approximate product.

Some scientific calculators allow you to multiply fractions. These calculators have the $\boxed{a\frac{b}{c}}$ key. Here is how these calculators work. (If the steps outlined in the next three examples do not work on your calculator, refer to the Owner's Manual for your calculator.)

Example 1: What is $\frac{1}{5} \times \frac{2}{3}$?

Key Press	Display	
1 $a\frac{b}{c}$ 5	1 ⌟ 5	← This means $\frac{1}{5}$.
× 2 $a\frac{b}{c}$ 3	2 ⌟ 3	← This means $\frac{2}{3}$.
=	2 ⌟ 15	← This means $\frac{2}{15}$.

So, $\frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$.

Example 2: What is $1\frac{6}{7} \times \frac{2}{3}$?

Key Press	Display	
1 $a\frac{b}{c}$ 6 $a\frac{b}{c}$ 7	1 ⌟ 6 ⌟ 7	← This means $1\frac{6}{7}$.
× 2 $a\frac{b}{c}$ 3	2 ⌟ 3	← This means $\frac{2}{3}$.
=	1 ⌟ 5 ⌟ 21	← This means $1\frac{5}{21}$.

So, $1\frac{6}{7} \times \frac{2}{3} = 1\frac{5}{21}$.

Example 3: What is $1\frac{2}{3} \times 2\frac{1}{5}$?

Key Press	Display	
1 $a\frac{b}{c}$ 2 $a\frac{b}{c}$ 3	1 ⌟ 2 ⌟ 3	← This means $1\frac{2}{3}$.
× 2 $a\frac{b}{c}$ 1 $a\frac{b}{c}$ 5	2 ⌟ 1 ⌟ 5	← This means $2\frac{1}{5}$.
=	3 ⌟ 2 ⌟ 3	← This means 3.

So, $1\frac{2}{3} \times 2\frac{1}{5} = 3\frac{2}{3}$.

Concluding Activities

Space for Your Work

1. Find the product for each of the following. Use a calculator which has the $a\frac{b}{c}$ key.

   a. $\frac{1}{2} \times \frac{7}{10}$

   b. $\frac{2}{3} \times \frac{1}{5}$

   c. $\frac{7}{9} \times \frac{3}{5}$

   d. $1\frac{4}{5} \times 1\frac{2}{3}$

   e. $2\frac{3}{8} \times 5\frac{1}{7}$

2. Danny was laying patio blocks. On Monday he laid $\frac{1}{3}$ of the blocks. On Tuesday he laid $\frac{1}{3}$ of the remaining blocks. On Wednesday he laid $\frac{1}{3}$ of the remaining blocks.

   a. If eight patio blocks were left, how many patio blocks were there to begin with?

   b. How many patio blocks did Danny lay on Monday?

   c. How many did he lay on Tuesday?

   d. How many blocks did Danny lay on Wednesday?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn cancelling techniques.

In this section you will learn this term.

- cancelling

Working Together

In Section 3 you learned to write equivalent fractions and to find basic fractions.

You learned that the identity element can be applied to fractions.

Example

- $\frac{3}{4} \times \boxed{1\ \frac{2}{2}} = \frac{6}{8}$
- $\frac{8}{12} \div \boxed{1\ \frac{4}{4}} = \frac{2}{3}$

Multiplying or dividing by 1 does not change the original value.

You can use the identity element to help you simplify the operation of multiplication as applied to fractions and mixed numbers.

Example 1: What is $\frac{2}{3} \times \frac{1}{2}$?

Solution

$$\frac{2}{3} \times \frac{1}{2}$$

$$= \frac{2 \times 1}{3 \times 2}$$

$$= \frac{\overset{1}{\cancel{2}} \times 1}{\cancel{2} \times 3}$$

$$= \frac{1 \times 1}{1 \times 3}$$

$$= \frac{1}{3}$$

The order in the denominator can be changed without changing the value since $\frac{2}{2} = 1$. Notice how the 2s are cancelled.

Example 2: What is $\frac{3}{4} \times \frac{4}{7}$?

Solution

$$\frac{3}{4} \times \frac{4}{7}$$

$$= \frac{3 \times 4}{4 \times 7}$$

$$= \frac{\overset{1}{\cancel{4}} \times 3}{\cancel{4} \times 7}$$

$$= \frac{1 \times 3}{1 \times 7}$$

$$= \frac{3}{7}$$

The order of the numerator can be changed without changing the value since $\frac{4}{4} = 1$. Notice how the 4s are cancelled.

Example 3: What is $1\frac{1}{5} \times 1\frac{1}{6}$?

Solution

$$1\frac{1}{5} \times 1\frac{1}{6}$$

$$= \frac{6}{5} \times \frac{7}{6}$$

$$= \frac{6 \times 7}{5 \times 6}$$

$$= \frac{\overset{1}{\cancel{6}} \times 7}{\cancel{6} \times 5}$$

$$= \frac{1 \times 7}{1 \times 5}$$

$$= \frac{7}{5}$$

$$= 1\frac{2}{5}$$

Change the mixed numbers to improper fractions.

Shift the order of the numbers in the denominator to show that $\frac{6}{6} = 1$.

The division of numerators and denominators by the same number is called **cancelling**. Cancelling can be done without shifting the numbers.

Example: What is $\frac{3}{5} \times \frac{1}{3}$?

Solution

$$\frac{3}{5} \times \frac{1}{3}$$

$$= \frac{\overset{1}{\cancel{3}} \times 1}{5 \times \cancel{3}}$$

$$= \frac{1}{5}$$

The numbers in the denominator do not have to be shifted before cancelling.

Now consider these examples.

Example 1: What is $\frac{3}{4} \times \frac{2}{7}$?

Solution

$$\frac{3}{4} \times \frac{2}{7}$$

$$= \frac{\cancel{2}^{1}}{\cancel{4}_{2} \times 7} \times 3$$

$$= \frac{3 \times 1}{2 \times 7}$$

$$= \frac{3}{14}$$

To cancel or simplify divide 2 and 4 by 2 to get $\frac{2}{4} = \frac{1}{4}$.

Example 2: What is $\frac{3}{4} \times \frac{2}{9}$?

Solution

$$\frac{3}{4} \times \frac{2}{9}$$

$$= \frac{\cancel{3}^{1} \times \cancel{2}^{1}}{\cancel{4}_{2} \times \cancel{9}_{3}}$$

$$= \frac{1 \times 1}{2 \times 3}$$

$$= \frac{1}{6}$$

To cancel or simplify divide 3 and 9 by 3 and divide 2 and 4 by 2 to get $\frac{3}{9} = \frac{1}{3}$ and $\frac{2}{4} = \frac{1}{2}$.

Example 3: What is $1\frac{7}{8} \times 2\frac{6}{7}$?

Solution

$$1\frac{7}{8} \times 2\frac{6}{7}$$

$$= \frac{15}{8} \times \frac{20}{7}$$

$$= \frac{15 \times \cancel{20}^{5}}{\cancel{8}_{2} \times 7}$$

$$= \frac{15 \times 5}{2 \times 7}$$

$$= \frac{75}{14}$$

$$= 5\frac{5}{14}$$

Change both mixed numbers to improper fractions.

To cancel divide 20 and 8 by 4 to get $\frac{20}{8} = \frac{5}{2}$.

Example 4: What is $1\frac{1}{5} \times 1\frac{1}{4}$?

Solution

$$1\frac{1}{5} \times 1\frac{1}{4}$$

$$= \frac{6}{5} \times \frac{5}{4}$$

$$= \frac{\cancel{6}^{3} \times \cancel{5}^{1}}{\cancel{5} \times \cancel{4}_{2}}$$

$$= \frac{3 \times 1}{1 \times 2}$$

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$

Change both mixed numbers to improper fractions.

To cancel divide 6 and 4 by 2 and divide 5 and 5 by 5 to get $\frac{6}{4} = \frac{3}{2}$ and $\frac{5}{5} = \frac{1}{1}$.

Practice Activities

Space for Your Work

Find each of these products. Use all the cancelling possible to simplify the answers.

1. $\frac{2}{3} \times \frac{3}{4}$

2. $\frac{3}{5} \times \frac{5}{8}$

3. $\frac{5}{8} \times \frac{4}{5}$

4. $\frac{2}{3} \times \frac{6}{7}$

5. $\frac{3}{4} \times \frac{7}{12}$

6. $\frac{8}{9} \times \frac{3}{4}$

7. $\frac{20}{39} \times \frac{13}{100}$

Space for Your Work

8. $1\frac{1}{6} \times \frac{3}{8}$

9. $\frac{4}{9} \times 1\frac{1}{2}$

10. $\frac{3}{5} \times 2\frac{2}{9}$

11. $3\frac{1}{3} \times 1\frac{1}{8}$

12. $2\frac{1}{4} \times 7\frac{3}{5}$

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

Computer Alternative

1. Do the multiplication program of the disk *Conquering Fractions (×, ÷)*.
   Press [S] to use the shortcuts.

Print Alternative

2. Find the product for each of the following. Use cancellation where possible. Express all final answers in simplest form.

   a. $\frac{3}{5} \times \frac{2}{3}$

   b. $\frac{3}{4} \times \frac{5}{6}$

   c. $\frac{5}{8} \times \frac{8}{25}$

   d. $\frac{5}{6} \times \frac{3}{4}$

e. $\frac{2}{5} \times 2\frac{1}{2}$

Space for Your Work

f. $3\frac{1}{3} \times \frac{9}{10}$

g. $1\frac{1}{3} \times 1\frac{1}{2}$

h. $1\frac{1}{3} \times 4\frac{1}{2}$

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Find the product for each of the following. Express all final answers in simplest form.

   a. $\frac{1}{2} \times 2$

   b. $3 \times \frac{1}{3}$

   c. $\frac{8}{5} \times \frac{5}{8}$

   d. $\frac{8}{9} \times \frac{8}{9}$

   e. $1\frac{1}{5} \times \frac{5}{6}$

   f. $\frac{3}{5} \times 1\frac{2}{3}$

   g. $2\frac{2}{3} \times \frac{3}{8}$

2. What pattern do you notice from Question 1? Each pair of numbers in Question 1 are **multiplicative inverses** or **reciprocals**.

Computer Alternative

Space for Your Work

3. Do Lesson 15 of the disk *Fractions* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA). If you need help, hold down the SHIFT key and press the [?] key.

Print Alternative

4. Give the multiplicative inverse or reciprocal for each of the following numbers.

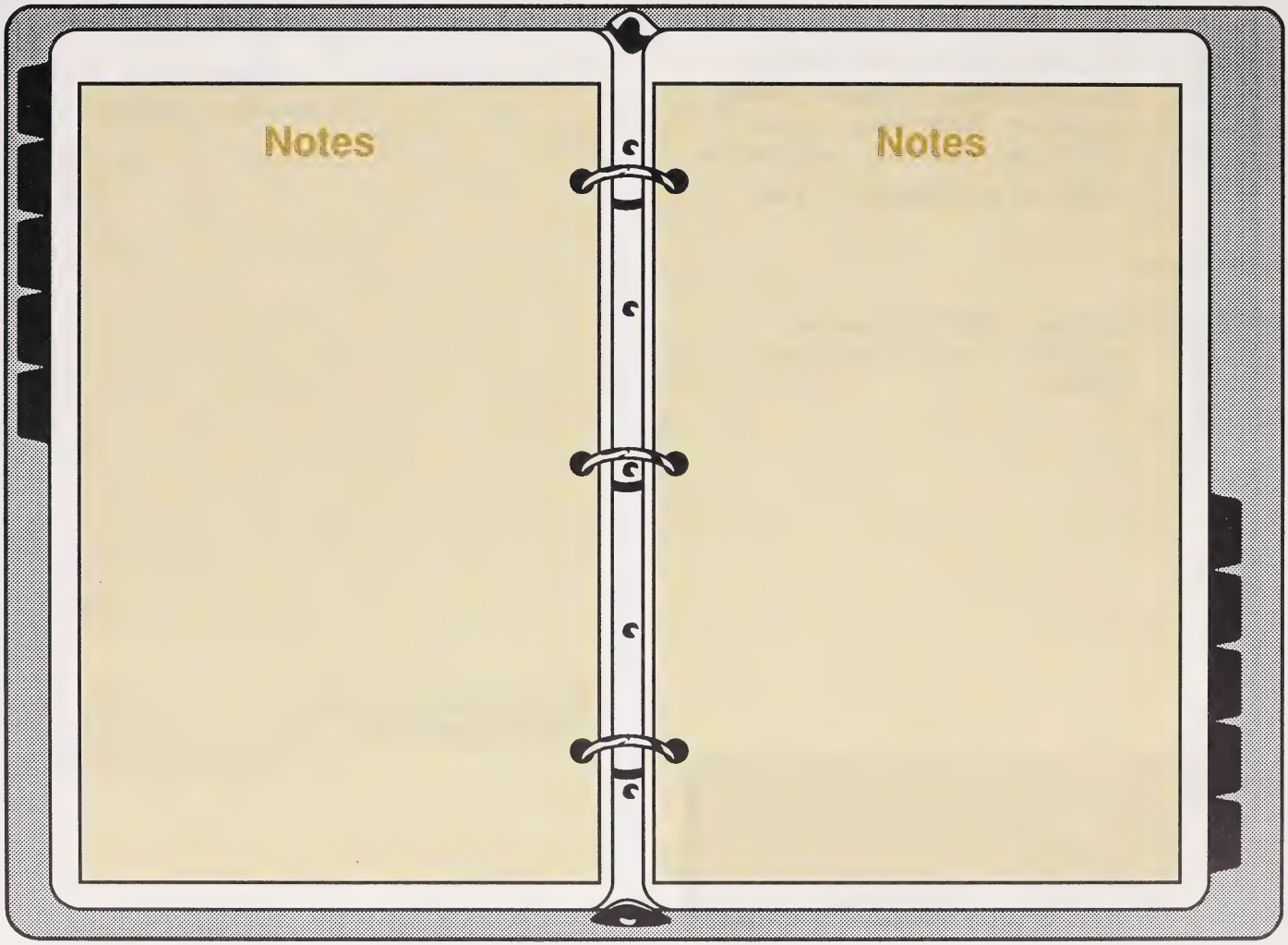
   a. $\frac{1}{7}$

   b. $\frac{3}{8}$

   c. 9

   d. $2\frac{1}{6}$

See your learning facilitator to check your answers and to receive further instructions.


Notes
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What Lies Ahead

In this section you will review strategies used to multiply whole numbers mentally and extend these strategies to include multiplying fractions and mixed numbers mentally.

Working Together

In previous schooling you have learned mental computation skills involving multiplication of whole numbers.

The first part of this section will review the methods of multiplying whole numbers mentally.

Video Activity

Watch the video *Solve It: Using Mental Computation for Multiplication* (AIT). If you cannot watch the video, read the notes which follow.

The Left-Right Method

Normally when you multiply, you multiply from right to left. When multiplying mentally, it is helpful to multiply from left to right using the distributive property.

Example

Think

$$321 \times 4 = (300 \times 4) + (20 \times 4) + (1 \times 4)$$
$$= 1200 + 80 + 4$$
$$= 1284$$

The Plus-Minus Method

It is sometimes difficult to multiply mentally when regrouping is necessary. In such cases, it helps to use the plus-minus method.

Example

Think

$$398 \times 4 = (400 \times 4) - (2 \times 4)$$
$$= 1600 - 8$$
$$= 1592$$

The Doubling-Halving Method

Another helpful method is to double one factor and halve the other factor. The objective is to get at least one factor to end with a zero.

Example

Think

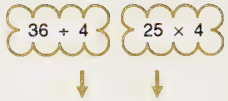
$26 \times 5 = 13 \times 10$

$= 130$

You can also quadruple one factor and quarter the other factor.

Example

Think



$36 \times 25 = 9 \times 100$

$= 900$

Introductory Activities

Space for Your Work

1. Multiply each of the following mentally using the left-right method.

   a. 123×3

   b. 512×4

   c. 934×2

2. Multiply each of the following mentally using the plus-minus method.

   a. 299×2

   b. 596×5

   c. 798×3

   d. 41×9

   e. 32×19

   f. 13×31

3. Multiply each of the following mentally.

Space for Your Work

a. 42×15

b. 124×25

See your learning facilitator to check your answers and to receive further instructions.

Working Together

The left-right method can sometimes be used to mentally multiply

- a mixed number and a whole number
- a mixed number and a fraction
- two mixed numbers

Example 1

Think

$$\begin{aligned} 4 \times 1\tfrac{1}{2} &= \left(4 \times 1\right) + \left(4 \times \tfrac{1}{2}\right) \\ &= 4 + 2 \\ &= 6 \end{aligned}$$

Example 2

Think

$$\begin{aligned} \tfrac{1}{3} \times 3\tfrac{1}{2} &= \left(\tfrac{1}{3} \times 3\right) + \left(\tfrac{1}{3} \times \tfrac{1}{6}\right) \\ &= 1 + \tfrac{1}{6} \\ &= 1\tfrac{1}{6} \end{aligned}$$

Example 3

Think

$$\begin{aligned} 1\tfrac{1}{2} \times 2\tfrac{1}{3} &= \tfrac{3}{2} \times 2\tfrac{1}{3} \\ &= \left(\tfrac{3}{2} \times 2\right) + \left(\tfrac{3}{2} \times \tfrac{1}{3}\right) \\ &= 3 + \tfrac{1}{2} \\ &= 3\tfrac{1}{2} \end{aligned}$$

Example 4

Think

$$\begin{aligned} 1\tfrac{1}{3} \times 3\tfrac{1}{4} &= \tfrac{4}{3} \times 3\tfrac{1}{4} \\ &= \left(\tfrac{4}{3} \times 3\right) + \left(\tfrac{4}{3} \times \tfrac{1}{4}\right) \\ &= 4 + \tfrac{1}{3} \\ &= 4\tfrac{1}{3} \end{aligned}$$

The plus-minus method can sometimes be helpful when multiplying fractions and mixed numbers.

Example 1

Think

$$\begin{aligned} 3 \times 1\tfrac{7}{8} &= 3 \times \left(1\tfrac{7}{8} + \tfrac{1}{8}\right) - 3 \times \tfrac{1}{8} \\ &= \left(3 \times 2\right) - \tfrac{3}{8} \\ &= 6 - \tfrac{3}{8} \\ &= 5\tfrac{5}{8} \end{aligned}$$

Example 2

Think

$$\begin{aligned} \tfrac{1}{4} \times 3\tfrac{1}{2} &= \tfrac{1}{4}\left(3\tfrac{1}{2} + \tfrac{1}{2}\right) - \tfrac{1}{4} \times \tfrac{1}{2} \\ &= \left(\tfrac{1}{4} \times 4\right) - \tfrac{1}{8} \\ &= 1 - \tfrac{1}{8} \\ &= \tfrac{7}{8} \end{aligned}$$

You can also use the doubling-halving method when multiplying fractions and mixed numbers. This means that you can double one factor and halve the other, triple one factor and third the other, quadruple one factor and quarter the other, and so on.

Example 1

Think

$$\begin{aligned} 1\tfrac{1}{2} \times 2\tfrac{2}{5} &= \left(2 \times 1\tfrac{1}{2}\right) \times \left(\tfrac{1}{2} \times 2\tfrac{2}{5}\right) \\ &= 3 \times 1\tfrac{1}{5} \\ &= 3 + \tfrac{3}{5} \\ &= 3\tfrac{3}{5} \end{aligned}$$

Example 2

Think

$$\begin{aligned} 2\tfrac{1}{3} \times \tfrac{3}{8} &= 3\left(2\tfrac{1}{3}\right) \times \left(\tfrac{1}{3} \times \tfrac{3}{8}\right) \\ &= \left(3 \times \tfrac{7}{3}\right) \times \left(\tfrac{1}{3} \times \tfrac{3}{8}\right) \\ &= 7 \times \tfrac{1}{8} \\ &= \tfrac{7}{8} \end{aligned}$$

Practice Activities

Space for Your Work

1. Mentally multiply each of the following using the left-right method.

   a. $3 \times 1\frac{2}{3}$

   b. $4 \times 2\frac{3}{4}$

   c. $\frac{1}{3} \times 6\frac{1}{2}$

   d. $\frac{1}{2} \times 4\frac{1}{5}$

2. Mentally multiply each of the following using the plus-minus method.

   a. $2 \times 1\frac{3}{4}$

   b. $3 \times 2\frac{4}{5}$

   c. $\frac{1}{2} \times 1\frac{7}{8}$

   d. $\frac{1}{4} \times 4\frac{4}{5}$

3. Mentally multiply each of the following using any method.

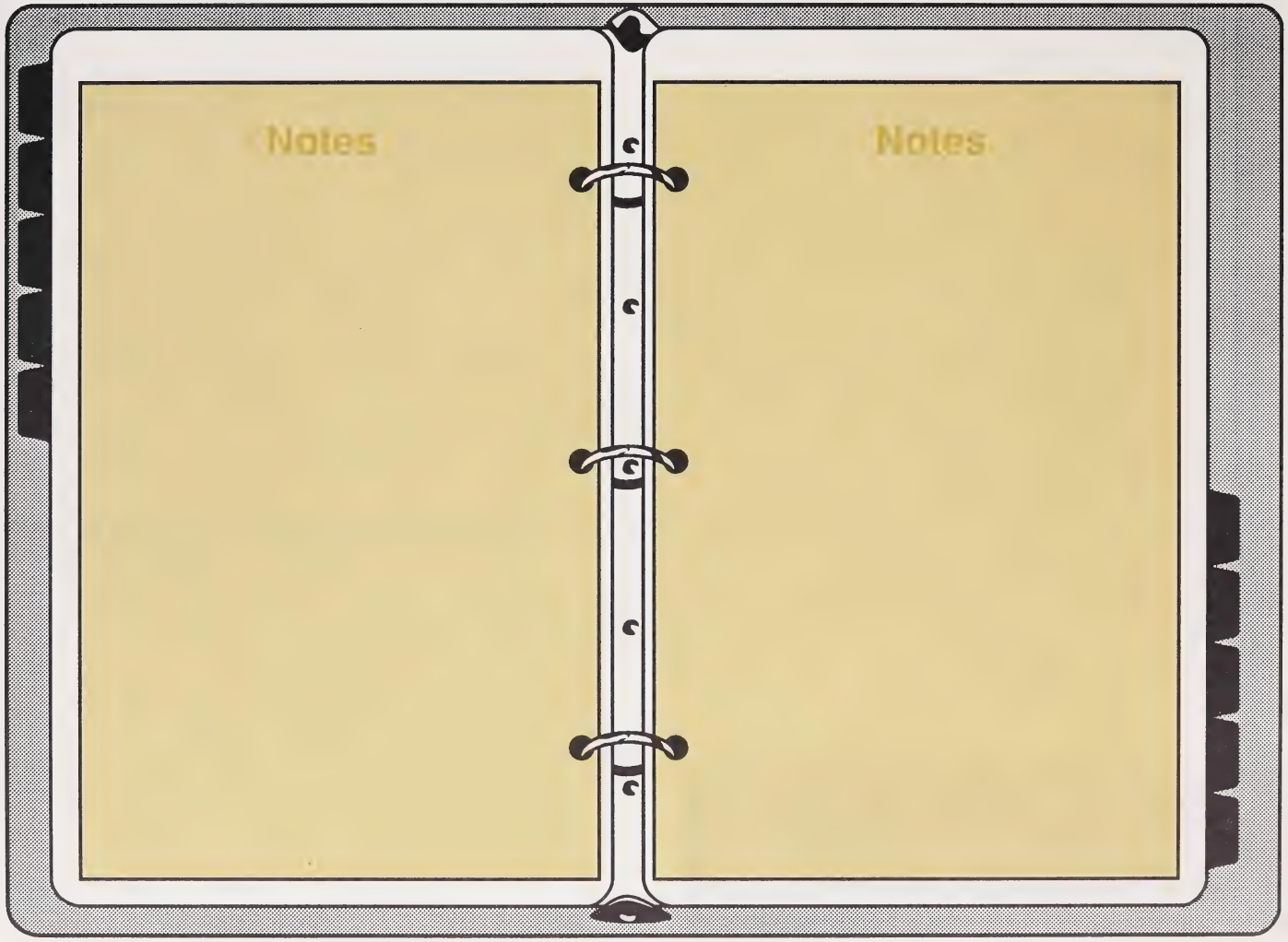
Space for Your Work

a. $\frac{1}{2} \times 2\frac{4}{5}$

b. $1\frac{1}{2} \times 4\frac{2}{3}$

c. $1\frac{2}{3} \times \frac{3}{4}$

See your learning facilitator to check your answers and to receive further instructions.


Notes
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What Lies Ahead

In this section you will learn these skills.

- estimating quotients
- diving whole numbers, fractions, and mixed numbers using one paper and pencil method.

Working Together

Division involves grouping.

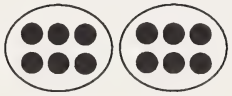
Example: What is 12 ÷ 2?

Solution

- 12 ÷ 2 can mean "How many groups of 2 are there in 12?"

There are six groups of 2 in 12.

- 12 ÷ 2 can mean "In 12 there are two groups of how many?"



There are two groups of 6 in 12.

Introductory Activities

Space for Your Work

1. Find the quotient for each of the following. Express all final answers in simplest form.

   a. $8 \div 10$

   b. $8 \div 9$

   c. $8 \div 8$

   d. $8 \div 4$

   e. $8 \div 2$

   f. $8 \div 1$

2. Use the results from Question 1 to make the following generalizations.

   a. If a number is divided by a number greater than itself, will the quotient be greater than 1, equal to 1, or less than 1?

   b. If a number is divided by itself, will the quotient be greater than 1, equal to 1, or less than 1.

   c. If a number is divided by a number less than itself, will the quotient be greater than 1, equal to 1, or less than 1?

   d. If a number is divided by 1, what will the quotient be?

3. Will the quotient of $8 \div \frac{1}{2}$ be greater than 8 or less than 8?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

More Division

Sometimes dividing whole numbers results in a whole number answer.

Example

Six chocolate bars are divided among three children. How many chocolate bars does each child get?

Solution

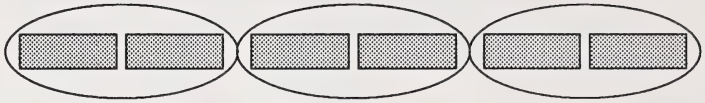
Estimate a value for $6 \div 3$. Because $6 \div 6 = 1$, and 3 is less than 6, the quotient will be greater than 1. Each child will get more than one bar.

Find the exact quotient.

$$6 \div 3 = 2$$

Each child will get two chocolate bars.

A diagram can be used to help you visualize the situation. Arrange the chocolate bars in three groups. How many bars are there in each group?



There are two bars in each group. This means that each child will get two chocolate bars.

Sometimes dividing whole numbers results in a mixed number.

Example

Four chocolate bars are shared by three children. How many chocolate bars does each child get?

Solution

Estimate a value for $4 \div 3$. Because $4 \div 4 = 1$, and 3 is less than 4, the quotient will be greater than 1.

Each child will get more than one bar.

Find the exact quotient.

Method 1

$$4 \div 3$$
$$= \frac{4}{3}$$
$$= 1\frac{1}{3}$$

$4 \div 3$ means $\frac{4}{3}$.

So each child gets $1\frac{1}{3}$ bars.

Method 2

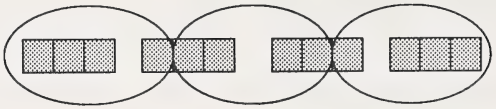
$$4 \div 3$$
$$= \frac{12}{3} \div \frac{3}{1}$$
$$= \frac{12 \div 3}{3 \div 1}$$
$$= \frac{4}{3}$$
$$= 1\frac{1}{3}$$

Divide the numerators. Then divide the denominators.

So, each child gets $1\frac{1}{3}$ bars.

A diagram can be used to help you visualize the situation.

Cut each bar into thirds and make three groups. How many thirds are there in each group?



Each child gets four-thirds or $\frac{4}{3}$ of a bar.

12 thirds ÷ 3 = 4 thirds

In other words, each child gets $1\frac{1}{3}$ bars.

When whole numbers are divided by fractions, the quotient will be larger than the dividend.

Example 1

Mrs. Robertson gives music lessons. Each lesson takes $\frac{3}{4}$ h. In six hours how many lessons can Mrs. Robertson give?

Solution

Estimate a value for $6 \div \frac{3}{4}$. Because $6 \div 1 = 6$, and $\frac{3}{4}$ is less than 1, $6 \div \frac{3}{4}$ will be greater than 6.

Mrs. Robertson can give more than six music lessons in six hours.

Find the exact quotient.

dividend → $6 \div \frac{3}{4}$ ← divisor

$= \frac{24}{4} \div \frac{3}{4}$

$= \frac{24 \div 3}{4 \div 4}$ ← Divide the numerators. Then divide the denominators.

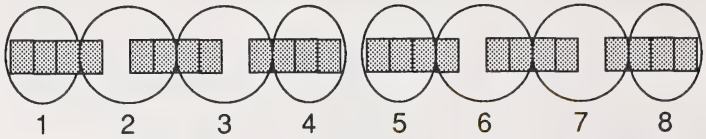
$= \frac{8}{1}$

$= 8$ ← quotient

Mrs. Robertson can give eight music lessons in six hours.

A diagram can be used to help you visualize the situation. Mrs. Robertson has six hours.

How many $\frac{3}{4}$ are there in 6? Divide each hour into fourths.



There are eight groups of $\frac{3}{4}$.

24 fourths ÷ 3 fourths = 8

So, Mrs. Robertson can give eight music lessons in six hours.

Example 2

On Tuesdays Mrs. Robertson has only four hours to give music lessons. How many $\frac{3}{4}$ h lessons can she give on Tuesday?

Solution

Estimate the value for $4 \div \frac{3}{4}$. Because $4 \div 1 = 4$, and $\frac{3}{4}$ is less than 1, $4 \div \frac{3}{4}$ will be greater than 4.

Mrs. Robertson can give more than four music lessons.

Find the exact quotient.

$$4 \div \frac{3}{4}$$

$$= \frac{16}{4} \div \frac{3}{4}$$

$$= \frac{16 \div 3}{4 \div 4}$$

$$= 5\frac{1}{3}$$

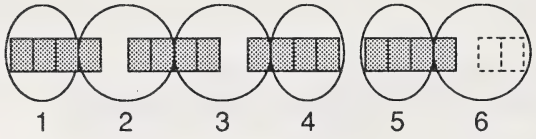
Divide the numerators. Then divide the denominators.

So, Mrs. Robertson can give $5\frac{1}{3}$ music lessons in four hours.

A diagram can be used to help you visualize the situation.

Mrs. Robertson has four hours.

How many $\frac{3}{4}$ are there in 4? Divide each hour into fourths. How many $\frac{3}{4}$ are there in $\frac{16}{4}$?



There are five whole groups and $\frac{1}{3}$ of a sixth group.

16 fourths ÷ 3 fourths = $5\frac{1}{3}$

So, Mrs. Robertson can give $5\frac{1}{3}$ music lessons in four hours.

When mixed numbers are divided by fractions, the quotient will be larger than the dividend.

Example 1

Hoi has $2\frac{2}{3}$ bags of peanuts for the party. If each bowl holds $\frac{2}{3}$ of a bag of peanuts, how many bowls are needed to serve all the peanuts?

Solution

Estimate the value for $2\frac{2}{3} \div \frac{2}{3}$. Since $2\frac{2}{3} \div 1 = 2\frac{2}{3}$, and $\frac{2}{3}$ is less than 1, the quotient is greater than $2\frac{2}{3}$.

Hoi needs more than $2\frac{2}{3}$ bowls.

Find the exact quotient.

dividend → $2\frac{2}{3} \div \frac{2}{3}$ ← divisor

$= \frac{8}{3} \div \frac{2}{3}$

$= \frac{8 \div 2}{3 \div 3}$ ← Divide the numerators. Then divide the denominators.

$= \frac{4}{1}$

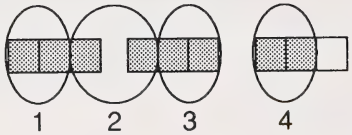
$= 4$ ← quotient

So, Hoi needs four bowls to serve $2\frac{2}{3}$ bags of peanuts.

A diagram can be used to help you visualize the situation.

Hoi has $2\frac{2}{3}$ bags of peanuts.

How many groups of $\frac{2}{3}$ are there in $2\frac{2}{3}$? Divide each whole bag into thirds. How many groups of $\frac{2}{3}$ are in $\frac{8}{3}$?



There are four equal groups.

8 thirds ÷ 2 thirds = 4

So, Hoi needs four bowls.

Example 2

Calen can build a model in $\frac{2}{3}$ of an hour.
How many models can he build in $1\frac{1}{2}$ h?

Solution

Estimate a value for $1\frac{1}{2} \div \frac{2}{3}$ or $\frac{3}{2} \div \frac{2}{3}$.

Since $1\frac{1}{2} \div 1 = 1\frac{1}{2}$ and $\frac{2}{3}$ is less than 1, $1\frac{1}{2} \div \frac{2}{3}$ is greater than $1\frac{1}{2}$.

So, Calen can build more than $1\frac{1}{2}$ models.

Find the exact quotient.

$$1\frac{1}{2} \div \frac{2}{3}$$

$$= \frac{3}{2} \div \frac{2}{3}$$

$$= \frac{9}{6} \div \frac{4}{6}$$

The LCM for 2 and 3 is 6.

$$= \frac{9 \div 4}{6 \div 6}$$

$$= 2\frac{1}{4}$$

So, Calen can build $2\frac{1}{4}$ models in $1\frac{1}{2}$ h.

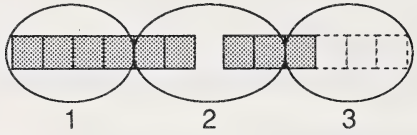
A diagram can be used to help you visualize the situation.

Calen has $1\frac{1}{2}$ hours.

How many $\frac{2}{3}$ are there in $1\frac{1}{2}$?

Divide the hours into sixths. This is the LCM for 2 and 3.

How many $\frac{4}{6}$ are there in $\frac{9}{6}$?



There are two whole groups and $\frac{1}{4}$ of a third group.

9 sixths ÷ 4 sixths = $2\frac{1}{4}$

So, Calen can build $2\frac{1}{4}$ models in $1\frac{1}{2}$ h.

When fractions are divided by fractions, compare the size of the dividend and the divisor. If the divisor is larger, the quotient will be greater than 1. If the dividend is larger, the quotient will be less than 1.

Example 1

Calen can build a model in $\frac{2}{3}$ of an hour. How many models can he build in $\frac{3}{4}$ of an hour?

Solution

Estimate a value for $\frac{3}{4} \div \frac{2}{3}$. Since $\frac{3}{4} \div \frac{3}{4} = 1$, and $\frac{2}{3}$ is less than $\frac{3}{4}$, the quotient will be greater than 1.

Find the exact quotient.

$$\frac{3}{4} \div \frac{2}{3}$$

$$= \frac{9}{12} \div \frac{8}{12}$$

The LCM for 4 and 3 is 12.

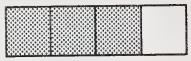
$$= \frac{9}{8}$$

$$= 1\frac{1}{8}$$

So, Calen can build $1\frac{1}{8}$ models in $\frac{3}{4}$ of an hour.

A diagram can be used to help you visualize the situation.

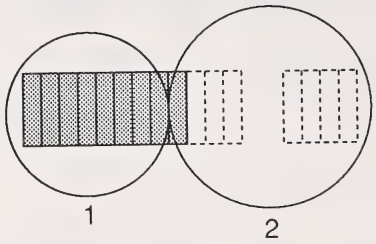
Calen has $\frac{3}{4}$ of an hour.



How many $\frac{2}{3}$ are there in $\frac{3}{4}$?

Divide the times into twelfths since the LCM for 3 and 4 is 12.

How many $\frac{8}{12}$ are there in $\frac{9}{12}$?



There is one whole group and $\frac{1}{8}$ of a second group.

9 twelfths ÷ 8 twelfths = $1\frac{1}{8}$

So, Calen can build $1\frac{1}{8}$ models in $\frac{3}{4}$ of an hour.

Example 2

A shelf in a grocery store will hold only $\frac{4}{5}$ of a large carton of cereal boxes. How many shelves will be needed to display $\frac{1}{2}$ of a large carton?

Solution

Estimate a value for $\frac{1}{2} \div \frac{4}{5}$. Since $\frac{1}{2} \div \frac{1}{2} = 1$, and since $\frac{4}{5}$ is greater than $\frac{1}{2}$, $\frac{1}{2} \div \frac{4}{5}$ will be less than 1.

So, less than one shelf is needed.

Find the exact quotient.

$$\frac{1}{2} \div \frac{4}{5}$$

$$= \frac{5}{10} \div \frac{8}{10}$$

$$= \frac{5 \div 8}{10 \div 10}$$

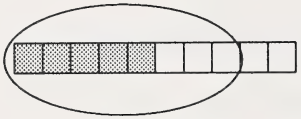
$$= \frac{5}{8}$$

To display $\frac{1}{2}$ of a carton of cereal boxes, $\frac{5}{8}$ of a shelf is needed.

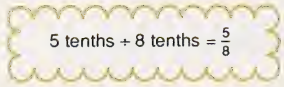
A diagram can be used to help you visualize the situation. The grocery clerk has $\frac{1}{2}$ of a carton of cereal boxes.

One shelf holds $\frac{4}{5}$ of a carton, so how many $\frac{4}{5}$ are there in $\frac{1}{2}$? Divide the carton into tenths since the LCM for 5 and 2 is 10.

How many $\frac{8}{10}$ are there in $\frac{5}{10}$?



There are $\frac{5}{8}$ of one whole group.



So, $\frac{5}{8}$ of a shelf is needed.

Practice Activities

Space for Your Work

1. Complete each of the following.

   a. eight-fifths ÷ 2

   b. six-tenths ÷ 3

   c. twelve-hundredths ÷ 4

   d. ten-fifteenths ÷ 5

2. Complete each of the following.

   a. eight-ninths ÷ two-ninths

   b. six-eighths ÷ three-eighths

   c. twelve-tenths ÷ four-tenths

   d. fifty-fourths ÷ five-fourths

   e. six-sevenths ÷ four-sevenths

   f. three-tenths ÷ seven-tenths

3. Find the quotient for each of the following. Express all final answers in simplest form.

Space for Your Work

a. $\frac{4}{5} \div 2$

b. $\frac{6}{7} \div 3$

c. $\frac{8}{9} \div 4$

d. $\frac{8}{9} \div \frac{2}{9}$

e. $\frac{6}{7} \div \frac{2}{7}$

f. $\frac{14}{10} \div \frac{7}{10}$

g. $\frac{3}{5} \div \frac{4}{5}$

h. $\frac{5}{9} \div \frac{2}{9}$

i. $\frac{3}{7} \div \frac{6}{7}$

4. Provide values for each of the following. Then find and simplify the final answer.

Space for Your Work

a. $\frac{3}{4} \div \frac{3}{8} = \frac{\square}{8} \div \frac{3}{8}$

b. $\frac{3}{4} \div \frac{1}{3} = \frac{\square}{12} \div \frac{\square}{12}$

c. $3 \div \frac{3}{7} = \frac{\square}{7} \div \frac{3}{7}$

d. $\frac{5}{6} \div \frac{2}{9} = \frac{\square}{18} \div \frac{\square}{18}$

e. $\frac{2}{3} \div \frac{5}{9} = \frac{\square}{9} \div \frac{5}{9}$

5. Find the quotient for each of the following. Simplify all final answers.

   a. $\frac{1}{2} \div \frac{1}{4}$

   b. $2 \div \frac{2}{5}$

   c. $\frac{2}{3} \div \frac{1}{6}$

   d. $2\frac{1}{2} \div \frac{5}{8}$

6. Joseph can swim once across the river in $\frac{1}{6}$ h. How many times can he cross the river in $\frac{3}{4}$ h?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

1. Complete each of the following.

   a. 5 sevenths ÷ 5

   b. 4 ninths ÷ 2

   c. 6 tenths ÷ 3

   d. 20 fifths ÷ 4

2. Complete each of the following.

   a. 6 sixths ÷ 2 sixths

   b. 6 ninths ÷ 3 ninths

   c. 10 twelfths ÷ 2 twelfths

   d. 8 halves ÷ 4 halves

   e. 7 eighths ÷ 5 eighths

   f. 3 fifths ÷ 4 fifths

3. Find the quotient for each of the following. Simplify all final answers.

   a. $\frac{4}{7} \div 2$

   b. $\frac{6}{11} \div 3$

c. $\frac{10}{13} \div 5$

d. $\frac{6}{6} \div \frac{2}{6}$

e. $\frac{6}{9} \div \frac{3}{9}$

f. $\frac{8}{7} \div \frac{4}{7}$

g. $\frac{5}{3} \div \frac{2}{3}$

h. $\frac{4}{5} \div \frac{3}{5}$

i. $\frac{2}{7} \div \frac{5}{7}$

4. Find the quotient for each of the following. Simplify all final answers.

a. $\frac{4}{8} \div \frac{1}{2}$

b. $4 \div \frac{3}{5}$

c. $3\frac{1}{2} \div \frac{3}{4}$

d. $\frac{1}{10} \div \frac{2}{5}$

e. $1\frac{1}{4} \div \frac{2}{3}$

Space for Your Work

5. Thurman wants to serve eight bags of chips at his party. If each of the bowls holds $\frac{2}{5}$ of a bag, how many bowls does he need to serve all the chips?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Accuracy with Fractions and Decimal Numbers

If you need to divide fractions that, when changed to decimal numbers, produce terminating decimal numbers, you can use either the fraction or the decimal number form to find the quotient.

Example: Find $\frac{3}{5} \div \frac{2}{5}$ using fractions and decimal numbers.

Solution

$$\frac{3}{5} \div \frac{2}{5}$$
$$= \frac{3 \div 2}{5 \div 5}$$
$$= \frac{3}{2}$$
$$= 1\frac{1}{2}$$

$$\frac{3}{5} = 0.6$$
$$\frac{2}{5} = 0.4$$
$$0.6 \div 0.4 = 1.5$$

$$1\frac{1}{2} = 1.5$$

Both $1\frac{1}{2}$ and 1.5 are exact quotients.

If you need to divide fractions that produce a repeating decimal number, you must use the fractions for an accurate answer. Using the decimal number forms results in an approximate quotient.

Example: Find $\frac{1}{2} \div \frac{1}{3}$ using fractions and decimal numbers.

Solution

$$\frac{1}{2} \div \frac{1}{3}$$
$$= \frac{3}{6} \div \frac{2}{6}$$
$$= \frac{3 \div 2}{6 \div 6}$$
$$= \frac{3}{2}$$
$$= 1\frac{1}{2}$$

$$\frac{1}{2} = 0.5$$
$$\frac{1}{3} \doteq 0.3$$
$$0.5 \div 0.3 \doteq 1.7$$

$$1\frac{1}{2} \neq 1.7$$

$1\frac{1}{2}$ is an exact quotient.

1.7 is an approximate quotient.

Most scientific calculators will perform the operation of division involving fractions and mixed numbers. If your calculator does not work as outlined below, refer to the manual for your calculator.

Example 1: What is $\frac{4}{9} \div \frac{7}{8}$?

Solution

Key Press	Display
4 $a\frac{b}{c}$ 9	4 ⌟ 9
÷ 7 $a\frac{b}{c}$ 8	? ⌟ 8
=	32 ⌟ 63

So, $\frac{4}{9} \div \frac{7}{8} = \frac{32}{63}$.

Example 2: What is $1\frac{5}{7} \div 2\frac{3}{4}$?

Solution

Key Press	Display
1 $a\frac{b}{c}$ 5 $a\frac{b}{c}$ 7	1 ⌟ 5 ⌟ ?
÷ 2 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 4	2 ⌟ 3 ⌟ 4
=	48 ⌟ ??

So, $1\frac{5}{7} \div 2\frac{3}{4} = \frac{48}{77}$.

Concluding Activities

Space for Your Work

1. Find the quotient for each of the following using the fraction form and the decimal number form. Compare the answers. If a difference occurs, tell which method is more accurate.

   a. $\frac{1}{2} \div \frac{3}{4}$

   b. $\frac{3}{10} \div \frac{2}{5}$

   c. $\frac{7}{9} \div \frac{2}{3}$

   d. $\frac{3}{7} \div \frac{4}{7}$

   e. $\frac{9}{20} \div \frac{7}{10}$

2. Evaluate each of the following by using a calculator which has a $\boxed{a\frac{b}{c}}$ key.

Space for Your Work

a. $\frac{3}{4} \div \frac{3}{8}$

b. $\frac{5}{12} \div \frac{1}{3}$

c. $2\frac{3}{4} \div \frac{9}{16}$

d. $1\frac{1}{5} \div 1\frac{1}{2}$

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

- changing division questions to multiplication questions
- dividing fractions and mixed numbers using a calculator

In this section you will learn these terms.

- reciprocal
- complex fraction

Working Together

In Section 12 you learned one paper and pencil method to divide fractions and mixed numbers. In this section you will learn another method to divide fractions and mixed numbers.

The method of dividing fractions that you will learn in this section uses reciprocals. In an earlier section you learned about reciprocals. **Reciprocals** are two numbers whose product is 1.

Example: Tell which numbers are reciprocals.

- $\frac{1}{3} \times \frac{3}{1} = 1$
- $\frac{4}{5} \times \frac{5}{4} = 1$

Solution

- Because $\frac{1}{3} \times \frac{3}{1} = 1$, $\frac{1}{3}$ is the reciprocal of $\frac{3}{1}$, and $\frac{3}{1}$ is the reciprocal of $\frac{1}{3}$.
- Because $\frac{4}{5} \times \frac{5}{4} = 1$, $\frac{4}{5}$ is the reciprocal of $\frac{5}{4}$, and $\frac{5}{4}$ is the reciprocal of $\frac{4}{5}$.

Dividing a number by itself gives a quotient of 1. This can be shown as follows.

- $\frac{1}{3} \div \frac{1}{3} = 1$
- $\frac{4}{5} \div \frac{4}{5} = 1$

Since $\frac{1}{3} \div \frac{1}{3} = 1$,

and $\frac{1}{3} \times 3 = 1$,

then $\frac{1}{3} \div \frac{1}{3} = \frac{1}{3} \times 3$.

Also, since $\frac{4}{5} \div \frac{4}{5} = 1$,

and $\frac{4}{5} \times \frac{5}{4} = 1$,

then $\frac{4}{5} \div \frac{4}{5} = \frac{4}{5} \times \frac{5}{4}$.

You can divide fractions and mixed numbers by changing the division question to a multiplication problem.

Example 1

Sharon has three balls of yarn. It takes $\frac{1}{2}$ a ball to knit one mitten. How many mittens can she knit if she were to use all the yarn that she has?

Solution

Estimate the value of $3 \div \frac{1}{2}$.

Since $3 \div \frac{1}{2} = 3$ and $\frac{1}{2}$ is less than 1, $3 \div \frac{1}{2}$ will be greater than 3.

Find the exact quotient.

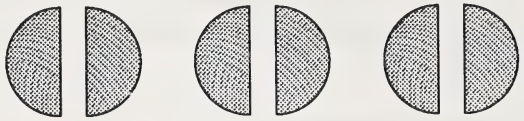
$$3 \div \frac{1}{2}$$
$$= 3 \times \frac{2}{1}$$ ← $\frac{2}{1}$ is the reciprocal of $\frac{1}{2}$
$$= \frac{6}{1}$$
$$= 6$$

Sharon can knit six mittens if she were to use the three balls of yarn.

A diagram can be used to help you visualize the situation.

Sharon has three balls of yarn.

In three balls of yarn how many groups of $\frac{1}{2}$ are there?



There are six groups of $\frac{1}{2}$.

Example 2

Nicki has six pizzas for a birthday party. She wants to serve $\frac{3}{8}$ of a pizza to each person. How many people can she feed by using the six pizzas?

Solution

Estimate the value of $6 \div \frac{3}{8}$.

Because $6 \div 1 = 6$, and $\frac{3}{8}$ is less than 1, $6 \div \frac{3}{8}$ will be greater than 6.

Calculate the exact answer.

$$6 \div \frac{3}{8}$$
$$= 6 \times \frac{8}{3}$$
$$= \frac{48}{3}$$
$$= 16$$

$\frac{8}{3}$ is the reciprocal of $\frac{3}{8}$

Nicki can feed 16 people using all six pizzas.

Simplifying techniques or cancelling can be used as shown next.

$$6 \div \frac{3}{8}$$
$$= \overset{2}{\cancel{6}} \times \frac{8}{\cancel{3}_{1}}$$
$$= 16$$

A diagram can be used to help you visualize the situation.

Nicki has six pizzas.

In 6 how many groups of $\frac{3}{8}$ each are there?

There are 16 groups of $\frac{3}{8}$ each.

Nicki can feed 16 people using all six pizzas..

Example 3

Mrs. Smith divided $\frac{1}{2}$ a pizza among her four children. How much of the whole pizza did each child receive?

Solution

Estimate the value of $\frac{1}{2} \div 4$.

Because $\frac{1}{2} \div 1 = \frac{1}{2}$ and 4 is greater than 1, $\frac{1}{2} \div 4$ will be less than $\frac{1}{2}$.

Find the exact quotient.

$$\frac{1}{2} \div 4$$

$$= \frac{1}{2} \times \frac{1}{4}$$ ← $\frac{1}{4}$ is the reciprocal of 4.

$$= \frac{1 \times 1}{2 \times 4}$$

$$= \frac{1}{8}$$

Mrs. Smith gave each of her four children $\frac{1}{8}$ of the whole pizza.

A diagram can be used to help you visualize the situation.

Mrs. Smith has $\frac{1}{2}$ a pizza. In $\frac{1}{2}$ there are four groups of how many?

There are four groups of $\frac{1}{8}$.

So, each of the four children receives $\frac{1}{8}$ of the whole pizza.

Example 4

Mr. MacCarthy has $\frac{3}{4}$ of a tank of gas. He can travel to his cottage and back once on $\frac{1}{4}$ of a tank of gas. How many return trips can he make on $\frac{3}{4}$ of a tank of gas?

Solution

Estimate the value of $\frac{3}{4} \div \frac{1}{4}$.

Because $\frac{3}{4} \div 1 = \frac{3}{4}$ and $\frac{1}{4}$ is less than 1, $\frac{3}{4} \div \frac{1}{4}$ will be more than $\frac{3}{4}$.

Find the exact quotient.

$$\frac{3}{4} \div \frac{1}{4}$$
$$= \frac{3}{4} \times \frac{4}{1}$$
$$= \frac{12}{4}$$
$$= 3$$

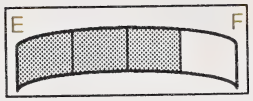
Cancelling can be used.

$$\frac{3}{4} \div \frac{1}{4}$$
$$= \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{1}$$
$$= 3$$

Mr. MacCarthy can make three return trips to the cottage on $\frac{3}{4}$ of a tank of gas.

A diagram can be used to help you visualize the situation.

Mr. MacCarthy has $\frac{3}{4}$ of a tank of gas. In $\frac{3}{4}$ how many groups of $\frac{1}{4}$ are there?



There are three groups of $\frac{1}{4}$ in $\frac{3}{4}$.

So, $\frac{3}{4} \div \frac{1}{4} = 3$.

Mr. MacCarthy can make three return trips to the cottage on $\frac{3}{4}$ of a tank of gas.

Practice Activites

Space for Your Work

1. Complete each of the following by providing the missing values. Find the quotient and express it in simplest form.

   a. $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{\square}{\square}$

   b. $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{\square}{\square}$

   c. $\frac{7}{5} \div 3 = \frac{7}{5} \times \frac{\square}{\square}$

   d. $4 \div 1\frac{2}{3} = 4 \div \frac{5}{3}$

   $= 4 \times \frac{\square}{\square}$

2. Change each of the following division problems to a multiplication problem and calculate each answer. Express all final answers in simplest form.

   a. $\frac{5}{8} \div \frac{3}{4}$

   b. $\frac{3}{5} \div \frac{9}{10}$

   c. $\frac{1}{2} \div 2$

   d. $\frac{3}{7} \div 4\frac{1}{5}$

   e. $3\frac{3}{4} \div 1\frac{1}{2}$

3. Bob takes $\frac{1}{10}$ h to complete one lap of a jogging track. How many laps can he complete in $\frac{3}{4}$ h?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

Do Question 1 or 2. Then do Questions 3 and 4.

Computer Alternative

1. a. Do Lessons 16, 17, 18, and 19 of the disk *Fractions* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA). If you need help, remember to hold down SHIFT key and press the [?] key.

   b. Do the division program on the *Conquering Fractions* disk from the MECC series. Press [S] to use the short cuts. Press [V] to view any of the problems in picture form.

Print Alternative

Space for Your Work

2. Find the quotients for each of the following. Express all final answers in simplest form.

a. $\frac{3}{4} \div \frac{3}{8}$

b. $\frac{3}{5} \div 3$

c. $8 \div \frac{4}{9}$

d. $1\frac{3}{8} \div 2\frac{3}{4}$

e. $3\frac{3}{4} \div 4\frac{1}{2}$

3. Yolande has $1\frac{1}{2}$ h to do six essay questions. About how long should she spend on each question?

4. Althea works in an automobile factory. It takes her $\frac{3}{4}$ h to assemble the seats for a car. How many seats can she assemble in a $1\frac{1}{2}$ h shift?

See your learning facilitator to check your answers and to receive further instructions.

Working Together

A **complex fraction** has a fraction in the numerator or in the denominator.

$\dfrac{\frac{1}{2}}{\frac{5}{6}}$ is a complex fraction.

You will remember that the line which separates the numerator from the denominator means division. This line is commonly called a **bar**. The proper name for this line is **vinculum**.

Notice the vinculum is made wider when there is a fraction in the numerator or in the denominator.

Complex fractions can be simplified as you will see here.

$$\frac{\frac{1}{2}}{\frac{5}{6}}$$

$$= \frac{1}{2} \div \frac{5}{6}$$

$$= \frac{1}{\cancel{2}_{1}} \times \frac{\cancel{6}^{3}}{5}$$

$$= \frac{3}{5}$$

Concluding Activities

Space for Your Work

Write each of these complex fractions as a division problem. Next, do the division and express the quotient in simplest form.

1. $\dfrac{\frac{2}{7}}{\frac{2}{9}}$

2. $\dfrac{\frac{1}{3}}{\frac{3}{4}}$

3. $\dfrac{2\frac{1}{2}}{3\frac{1}{4}}$

4. $\dfrac{7}{3\frac{1}{2}}$

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will review strategies for dividing whole numbers mentally and extend these strategies to include the division of fractions and the division of mixed numbers.

Working Together

Previously in mathematics you learned mental computation skills involving division of whole numbers.

The first part of this section will review these methods of dividing whole numbers mentally.

The Left-Right Method

If regrouping is not needed, you can divide from left to right.

Example 1: What is 369 ÷ 3?

Solution

Think $369 \div 3$

$= (300 \div 3) + (60 \div 3) + (9 \div 3)$

$= 100 + 20 + 3$

$= 123$

Example 2: What is 480 ÷ 4?

Solution

Think $480 \div 4$

$= (400 \div 4) + (80 \div 4)$

$= 100 + 20$

$= 120$

Example 3: What is 215 ÷ 5?

Solution

Think $215 \div 5$

$= (200 \div 5) + (15 \div 5)$

$= 40 + 3$

$= 43$

Example 4: What is 147 ÷ 7?

Solution

Think $147 \div 7$

$= (140 \div 7) + (7 \div 7)$

$= 20 + 1$

$= 21$

The Plus-Minus Method

Sometimes the plus-minus method can be helpful.

Example 1: What is $414 \div 6$?

Solution

Think $414 \div 6$

$= (420 - 6) \div 6$

$= (420 \div 6) - (6 \div 6)$

$= 70 - 1$

$= 69$

Example 2: What is $492 \div 4$?

Solution

Think $492 \div 4$

$= (480 + 12) \div 4$

$= (480 \div 4) + (12 \div 4)$

$= 120 + 3$

$= 123$

The Doubling-Tripling-Quadrupling Method

Sometimes the doubling method for division is helpful.

Example 1: What is $60 \div 5$?

Solution

Think $60 \div 5$

$= (60 \times 2) \div (5 \times 2)$

$= 120 \div 10$

$= 12$

You can also triple or quadruple.

Example 2: What is $400 \div 25$?

Solution

Think $400 \div 25$

$= (400 \times 4) \div (25 \times 4)$

$= 1600 \div 100$

$= 16$

Introductory Activities

Space for Your Work

1. Compute each of the following mentally. Use the left-right method.
   a. 963 ÷ 3
   b. 425 ÷ 5
   c. 372 ÷ 6

2. Compute each of the following mentally. Use the plus-minus method.
   a. 396 ÷ 4
   b. 496 ÷ 4
   c. 1008 ÷ 9

3. Compute each of the following mentally. Use the doubling-tripling-quadrupling method.
   a. 600 ÷ 25
   b. 325 ÷ 5
   c. 630 ÷ 15

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Finding Common Denominators

In some division problems it is easy to divide using the common denominator method.

Example 1: What is $\frac{3}{4} \div \frac{3}{8}$?

Solution

Think $\frac{3}{4} \div \frac{3}{8}$

$= \frac{6}{8} \div \frac{3}{8}$

$= \frac{2}{1}$

$= 2$

Divide the numerators. Then divide the denominators.

Example 2: What is $\frac{9}{10} \div \frac{2}{5}$?

Solution

Think $\frac{9}{10} \div \frac{2}{5}$

$= \frac{9}{10} \div \frac{4}{10}$

$= \frac{9}{4}$

$= 2\frac{1}{4}$

Divide the numerators. Then divide the denominators.

Multiplying by the Reciprocal

In other division it is easier to multiply the dividend by the reciprocal of the divisor.

Example 1: What is $\frac{3}{5} \div \frac{1}{2}$?

Solution

Think $\frac{3}{5} \div \frac{1}{2}$

$= \frac{3}{5} \times \frac{2}{1}$

$= \frac{6}{5}$

$= 1\frac{1}{5}$

Multiply the numerators. Then multiply the denominators.

Example 2: What is $\frac{2}{3} \div \frac{1}{8}$?

Solution

Think $\frac{2}{3} \div \frac{1}{8}$

$= \frac{2}{3} \times \frac{8}{1}$

$= \frac{16}{3}$

$= 5\frac{1}{3}$

Multiply the numerators. Then multiply the denominators.

The Left-Right Method

It is sometimes helpful to use the left-right method when dividing mixed numbers.

Example 1: What is $3\frac{3}{5} \div 3$?

Solution

Think $3\frac{3}{5} \div 3$

$= \left(3 \div 3\right) + \left(\frac{3}{5} \div 3\right)$

$= 1 + \frac{1}{5}$

$= 1\frac{1}{5}$

Example 2: What is $1\frac{1}{2} \div \frac{1}{4}$?

Solution

Think $1\frac{1}{2} \div \frac{1}{4}$

$= \left(1 \div \frac{1}{4}\right) + \left(\frac{1}{2} \div \frac{1}{4}\right)$

$= 4 + 2$

$= 6$

The Plus-Minus Method

It is sometimes helpful to use the plus-minus method when dividing mixed numbers and whole numbers.

Example 1: What is $4\frac{3}{4} \div 5$?

Solution

Think $4\frac{3}{4} \div 5$

$= \left(5 \div 5\right) - \left(\frac{1}{4} \div 5\right)$

$= 1 - \frac{1}{20}$

$= \frac{19}{20}$

Example 2: What is $3\frac{4}{7} \div 3$?

Solution

Think $3\frac{4}{7} \div 3$

$= \left(3\frac{3}{7} \div 3\right) + \left(\frac{1}{7} \div 3\right)$

$= 1\frac{1}{7} + \frac{1}{21}$

$= 1\frac{4}{21}$ ← $1\frac{3}{21} + \frac{1}{21}$

The Doubling-Tripling-Quadrupling Method

Sometimes the doubling method is helpful when dividing fractions and mixed numbers.

Example 1: What is $1\frac{1}{2} \div \frac{1}{2}$?

Solution

Think $1\frac{1}{2} \div \frac{1}{2}$

$= \left(1\frac{1}{2} \times 2\right) \div \left(\frac{1}{2} \times 2\right)$

$= 3 \div 1$

$= 3$

Example 2: What is $3 \div 1\frac{1}{2}$?

Solution

Think $3 \div 1\frac{1}{2}$

$= \left(3 \times 2\right) \div \left(1\frac{1}{2} \times 2\right)$

$= 6 \div 3$

$= \left(6 \div 3\right)$

$= 2$

You may also triple, quadruple, and so on.

Example 1: What is $4 \div 1\frac{1}{3}$?

Solution

Think $4 \div 1\frac{1}{3}$

$= \left(4 \times 3\right) \div \left(1\frac{1}{3} \times 3\right)$

$= 12 \div 4$

$= 3$

Example 2: What is $2\frac{1}{2} \div \frac{1}{4}$?

Solution

Think $\frac{1}{2} \div \frac{1}{4}$

$= \left(2 \times 4\right) \div \left(\frac{1}{4} \times 4\right)$

$= 8 \div 1$

$= 8$

Practice Activities

Space for Your Work

1. Mentally divide each of the following. Use common denominators.

   a. $\frac{15}{16} \div \frac{3}{8}$

   b. $\frac{5}{12} \div \frac{5}{6}$

   c. $8 \div \frac{1}{3}$

2. Mentally divide each of the following. Use reciprocals.

   a. $11 \div 1\frac{3}{8}$

   b. $\frac{3}{7} \div \frac{1}{2}$

   c. $\frac{2}{3} \div 4$

3. Divide each of the following mentally. Use the right-left method.

   a. $6\frac{4}{7} \div 2$

   b. $9\frac{6}{11} \div 3$

   c. $1\frac{1}{2} \div \frac{1}{4}$

4. Mentally divide each of the following. Use the plus-minus method.

   a. $2\frac{2}{3} \div \frac{1}{3}$

   b. $7\frac{1}{4} \div 4$

   c. $1\frac{2}{7} \div 2$

5. Mentally divide each of the following. Use the doubling-tripling-quadrupling method.

   a. $3\frac{3}{8} \div 1\frac{1}{2}$

   b. $3\frac{3}{4} \div \frac{3}{4}$

   c. $4\frac{4}{5} \div 1\frac{1}{3}$

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. The quotient of two numbers is less than 1. Find the quotient if the numbers are $\frac{1}{2}$ and $\frac{2}{5}$.

2. The quotient of two numbers is greater than 1. Find the quotient if the numbers are $\frac{2}{5}$ and $\frac{3}{5}$.

3. The quotient of a larger number and a smaller number is $\frac{1}{2}$. What is the smaller number if the larger number is $\frac{3}{4}$?

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

- estimating the sum of a series of addends
- using short cuts to find the exact sum

Working Together

In Sections 6 to 14 you performed operations on fractions and mixed numbers. However, you worked with only two numbers at a time.

In this section you will examine a series of sums.

Consider this example.

Example 1

Judith kept track of the number of hours she spent babysitting in April.

April 6	$3\frac{1}{4}$ h
April 14	$5\frac{1}{2}$ h
April 19	$2\frac{3}{4}$ h
April 20	$4\frac{1}{3}$ h
April 29	$2\frac{1}{5}$ h

In all, about how many hours did Judith spend babysitting during April?

Solution

- Estimate by rounding. Round to the nearest unit or one.

$$\begin{array}{rcr} 3\frac{1}{4} & \doteq & 3 \\ 5\frac{1}{2} & \doteq & 6 \\ 2\frac{3}{4} & \doteq & 3 \\ 4\frac{1}{3} & \doteq & 4 \\ +\,2\frac{1}{5} & \doteq & +\,2 \\ \hline & & 18 \end{array}$$

Judith babysat for about 18 hours in April.

- Estimating by using betweenness.

Rounding down			Rounding up		
$3\frac{1}{4}$	$\doteq$	3	$3\frac{1}{4}$	$\doteq$	4
$5\frac{1}{2}$	$\doteq$	5	$5\frac{1}{2}$	$\doteq$	6
$2\frac{3}{4}$	$\doteq$	2	$2\frac{3}{4}$	$\doteq$	3
$4\frac{1}{3}$	$\doteq$	4	$4\frac{1}{3}$	$\doteq$	5
$+\,2\frac{1}{5}$	$\doteq$	$+\,2$	$+\,2\frac{1}{5}$	$\doteq$	$+\,3$
		16			21

Judith babysat between 16 h and 21 h during the month of April.

- Estimating using front-end digits.

$$\begin{array}{rcr} 3\frac{1}{4} & \doteq & 3 \\ 5\frac{1}{2} & \doteq & 5 \\ 2\frac{3}{4} & \doteq & 2 \\ 4\frac{1}{3} & \doteq & 4 \\ +\,2\frac{1}{5} & \doteq & +\,2 \\ \hline & & 16 \end{array}$$

Judith babysat over 16 h during the month of April.

Note

This is a low estimate as only the whole numbers were considered and used.

Example 2

Richard had a pizza party. He cut one pizza into eight equal pieces. He cut another pizza into six equal pieces and a final pizza into four equal pieces. Bill ate $\frac{5}{6}$ of a pizza. Richard ate $\frac{5}{8}$ of a pizza. Ruth ate $\frac{1}{4}$ of a pizza. Yvonne ate $\frac{1}{4}$ of a pizza. How much pizza did they eat altogether?

Solution

- Estimating by rounding.

$$\frac{5}{6} + \frac{5}{8} + \frac{1}{4} + \frac{1}{4}$$

$$\doteq 1 + 1 + 0 + 0$$

$$\doteq 2$$

The four party participants ate about two pizzas.

- Estimating using betweenness.

Rounding down	Rounding up
$\frac{5}{6} \doteq 0$	$\frac{5}{6} \doteq 1$
$\frac{5}{8} \doteq 0$	$\frac{5}{8} \doteq 1$
$\frac{1}{4} \doteq 0$	$\frac{1}{4} \doteq 1$
$+\frac{1}{4} \doteq +0$	$+\frac{1}{4} \doteq +1$
0	4

The four party participants ate between zero and four pizzas.

Note

Front-end digits estimating cannot be used in this case.

Refining Your Estimation

When estimating it is helpful to round fractions and mixed numbers to the nearest half.

Example 1: Round each of these fractions to the nearest $\frac{1}{2}$ and find the approximate sum.

$\frac{1}{3}, \frac{3}{4}, \frac{1}{2}, \frac{3}{5}, \frac{9}{10}, \frac{3}{10}$

Solution

$$\frac{1}{3} + \frac{3}{4} + \frac{1}{2} + \frac{3}{5} + \frac{9}{10} + \frac{3}{10}$$

$$\doteq \frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{2} + 1 + \frac{1}{2}$$

$$\doteq 4$$

Example 2: Round each of these fractions to the nearest $\frac{1}{2}$ and find the approximate sum.

$\frac{3}{4}, \frac{1}{12}, \frac{2}{3}, \frac{5}{6}, \frac{3}{8}, \frac{5}{12}$

Solution

$$\frac{3}{4} + \frac{1}{12} + \frac{2}{3} + \frac{5}{6} + \frac{3}{8} + \frac{5}{12}$$

$$\doteq 1 + 0 + \frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{2}$$

$$\doteq 3\frac{1}{2}$$

Example 3: Round each of these mixed numbers to the nearest $\frac{1}{2}$ and find the approximate sum.

$1\frac{1}{3}, 2\frac{5}{9}, 1\frac{5}{6}, 3\frac{2}{3}, 2\frac{7}{9}, 1\frac{1}{4}$

Solution

$$1\frac{1}{3} + 2\frac{5}{9} + 1\frac{5}{6} + 3\frac{2}{3} + 2\frac{7}{9} + 1\frac{1}{4}$$

$$\doteq 1\frac{1}{2} + 2\frac{1}{2} + 2 + 3\frac{1}{2} + 3 + 1\frac{1}{2}$$

$$\doteq 14$$

Example 4: Round each of these mixed numbers to the nearest $\frac{1}{2}$ and find the approximate sum.

$2\frac{5}{16}, 1\frac{5}{8}, 1\frac{1}{4}, 2\frac{1}{2}, 2\frac{3}{7}, 1\frac{3}{5}$

Solution

$$2\frac{5}{16} + 1\frac{5}{8} + 1\frac{1}{4} + 2\frac{1}{2} + 2\frac{3}{7} + 1\frac{3}{5}$$

$$\doteq 2 + 1\frac{1}{2} + 1\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2}$$

$$\doteq 11\frac{1}{2}$$

Introductory Activities

Space for Your Work

Estimate each of these sums. Round each addend to the nearest $\frac{1}{2}$ before adding.

1. $5\frac{1}{8} + 6\frac{3}{4} + 7\frac{1}{2}$

2. $1\frac{7}{8} + 2\frac{1}{9} + 3\frac{3}{5}$

3. $\frac{1}{2} + \frac{7}{8} + \frac{3}{7} + \frac{1}{6}$

4. $\frac{3}{4} + \frac{1}{5} + \frac{2}{3} + \frac{4}{9}$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Adding a Series of Fractions

Just as with whole numbers, the sum of several fractions is the same no matter how you choose to group them.

Example: What is the sum of $\frac{1}{9} + \frac{3}{5} + \frac{1}{2} + \frac{8}{9} + \frac{2}{5}$?

Solution

Estimate the sum by rounding each addend to the nearest $\frac{1}{2}$.

$$\frac{1}{9} + \frac{3}{5} + \frac{1}{2} + \frac{8}{9} + \frac{2}{5}$$

$$\doteq 0 + \frac{1}{2} + \frac{1}{2} + 1 + \frac{1}{2}$$

$$\doteq 2\frac{1}{2}$$

Calculate the exact sum. To do this, two methods can be used. Study both of them carefully.

Method 1

Add two fractions at a time, doing your work from left to right.

$$\frac{1}{9} + \frac{3}{5} + \frac{1}{2} + \frac{8}{9} + \frac{2}{5}$$

$$= \left(\frac{5}{45} + \frac{27}{45}\right) + \frac{1}{2} + \frac{8}{9} + \frac{2}{5}$$

$$= \frac{32}{45} + \frac{1}{2} + \frac{8}{9} + \frac{2}{5}$$

$$= \left(\frac{64}{90} + \frac{45}{90}\right) + \frac{8}{9} + \frac{2}{5}$$

$$= \frac{109}{90} + \frac{8}{9} + \frac{2}{5}$$

$$= \left(\frac{109}{90} + \frac{80}{90}\right) + \frac{2}{5}$$

$$= \frac{189}{90} + \frac{2}{5}$$

$$= \frac{189}{90} + \frac{36}{90}$$

$$= \frac{225}{90}$$

$$= 2\frac{45}{90}$$

$$= 2\frac{1}{2}$$

Method 2

You can change the fractions to fractions with common denominators and then add, working from left to right.

$$\frac{1}{9} + \frac{3}{5} + \frac{1}{2} + \frac{8}{9} + \frac{2}{5}$$

$$= \frac{10}{90} + \frac{54}{90} + \frac{45}{90} + \frac{80}{90} + \frac{36}{90}$$

$$= \frac{225}{90}$$

$$= 2\frac{45}{90}$$

$$= 2\frac{1}{2}$$

Method 3

You can add two fractions at a time in an order which is different from the order in which they were originally written.

$$\frac{1}{9} + \frac{3}{5} + \frac{1}{2} + \frac{8}{9} + \frac{2}{5}$$

$$= \left(\frac{1}{9} + \frac{8}{9}\right) + \left(\frac{3}{5} + \frac{2}{5}\right) + \frac{1}{2}$$

$$= \frac{9}{9} + \frac{5}{5} + \frac{1}{2}$$

$$= 1 + 1 + \frac{1}{2}$$

$$= 2\frac{1}{2}$$

From this method you can see that adding using the new order is much easier.

Practice Activities

Space for Your Work

For each of the following sums

- estimate the answer by rounding each addend to the nearest $\frac{1}{2}$.
- find the exact answer by using the easiest method.
- look for shortcuts and express each final answer in simplest form.

1. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{1}{3} + \frac{1}{4}$
2. $\frac{3}{4} + \frac{1}{3} + \frac{2}{5} + \frac{3}{5} + \frac{2}{3}$
3. $\frac{2}{5} + \frac{1}{5} + \frac{3}{10} + \frac{1}{2} + \frac{1}{3}$
4. $1\frac{1}{3} + 2\frac{1}{2} + 4\frac{1}{3} + 1\frac{1}{2}$
5. $6\frac{2}{3} + 4\frac{1}{2} + 7\frac{2}{3} + 1\frac{1}{2}$
6. $7\frac{1}{3} + 2\frac{1}{2} + 3\frac{2}{3} + 1\frac{3}{4} + 2\frac{2}{3}$

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

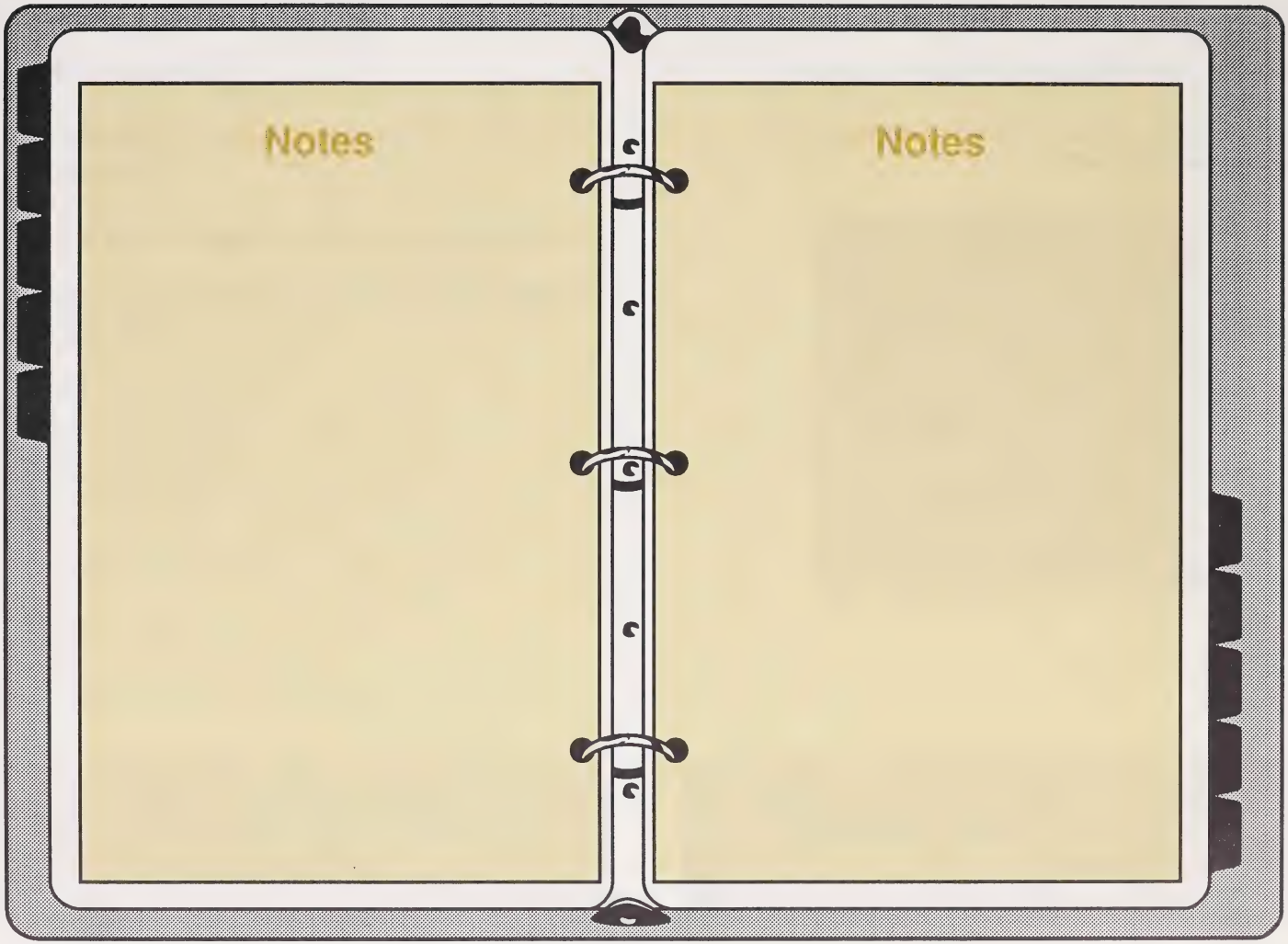
Space for Your Work

The magic sum for this partial magic square is $2\frac{1}{10}$.

Find the missing values which will make the square a complete magic square.

$\frac{3}{4}$	$\frac{1}{2}$		$\frac{3}{20}$
		$\frac{2}{5}$	$\frac{17}{20}$
	$\frac{13}{20}$		
$\frac{9}{10}$	$\frac{7}{20}$	$\frac{11}{20}$	

See your learning facilitator to check your answers and to receive further instructions.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What Lies Ahead

In this section you will learn these skills.

- estimating the product of a series of factors
- using shortcuts to find the exact product

Working Together

In the last section you examined and worked with a series of sums. In this section you will work with a series of products.

Estimating Products

When you are estimating the product of a series of mixed numbers, it is helpful to round each of the mixed numbers to the nearest whole number.

Example: Estimate the product for $2\frac{4}{5} \times 3\frac{3}{4} \times 1\frac{1}{5}$.

Solution

Think $2\frac{4}{5} \times 3\frac{3}{4} \times 1\frac{1}{5}$

$\doteq 3 \times 4 \times 1$

$\doteq 12$

When you are estimating the product of a series where the factors are a mixture of mixed numbers and fractions, do not round the fractions. It is best to round the mixed numbers only and then estimate the product.

Example: Estimate the product for $\frac{1}{5} \times 8\frac{1}{3} \times 2\frac{4}{5}$.

Solution

Think $\frac{1}{5} \times 8\frac{1}{3} \times 2\frac{4}{5}$

$\doteq \frac{1}{5} \times 8 \times 3$

$\doteq \frac{1}{5} \times 24$

$\doteq \frac{24}{5}$

$\doteq 5$

When you are estimating the product of a series of fractions, rounding is not helpful.

Example 1: What is $\frac{4}{5} \times \frac{2}{3} \times \frac{4}{7}$?

Solution

Think $1 \times \frac{4}{7} = \frac{4}{7}$

Since $\frac{4}{5} \times \frac{2}{3}$ is less than 1, $\frac{4}{5} \times \frac{2}{3} \times \frac{4}{7}$ is less than $\frac{4}{7}$.

Example 2: What is $\frac{1}{5} \times \frac{2}{7} \times \frac{3}{4}$?

Solution

Think $1 \times \frac{3}{4} = \frac{3}{4}$

Since $\frac{1}{5} \times \frac{2}{7}$ is less than 1, $\frac{1}{5} \times \frac{2}{7} \times \frac{3}{4}$ is less than $\frac{3}{4}$.

Introductory Activities

Space for Your Work

Estimate a product for each of the following.

1. $3\frac{3}{5} \times 5\frac{1}{4} \times 2\frac{1}{2}$
2. $5\frac{1}{7} \times 3\frac{2}{3} \times 4\frac{1}{4}$
3. $3\frac{5}{8} \times 7\frac{1}{6} \times 2\frac{4}{5}$
4. $\frac{1}{8} \times 2\frac{1}{2} \times 3\frac{3}{4}$
5. $\frac{1}{3} \times \frac{1}{2} \times 1\frac{1}{2}$
6. $\frac{1}{4} \times 4\frac{1}{5} \times 1\frac{1}{4}$
7. $\frac{1}{3} \times \frac{1}{8} \times \frac{1}{10}$
8. $\frac{7}{8} \times \frac{4}{5} \times \frac{3}{4}$
9. $\frac{1}{2} \times \frac{3}{7} \times \frac{5}{8}$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Evaluating the Product of a Series

When you multiply a series of fractions and mixed numbers, the order in which the multiplication is done does not matter. Moreover, cancelling can be used to simplify the multiplication and the final product.

Example: What is $\frac{6}{7} \times \frac{4}{5} \times \frac{5}{7}$?

Solution

You can multiply the fractions, two at a time, going from left to right.

$$\frac{6}{7} \times \frac{4}{5} \times \frac{5}{7}$$

$$= \frac{6 \times 4}{7 \times 5} \times \frac{5}{7}$$

$$= \frac{24}{\cancel{35}^{7}} \times \frac{\cancel{5}^{1}}{7}$$

$$= \frac{24}{49}$$

You can also rearrange the factors to make the multiplication easier.

$$\frac{6}{7} \times \frac{4}{5} \times \frac{5}{7}$$

$$= \frac{5}{7} \times \frac{4}{5} \times \frac{6}{7}$$

$$= \frac{\cancel{5}^{1} \times 4}{7 \times \cancel{5}_{1}} \times \frac{6}{7}$$

$$= \frac{24}{49}$$

Cancellation can be done without a rearrangement of any kind.

$$\frac{6}{7} \times \frac{4}{5} \times \frac{5}{7}$$

$$= \frac{6 \times 4 \times \cancel{5}^{1}}{7 \times \cancel{5}_{1} \times 7}$$

$$= \frac{24}{49}$$

Practice Activities

Space for Your Work

For each of the following

- estimate a product
- calculate the exact product
- use shortcuts or cancelling where possible

1. $1\frac{1}{2} \times \frac{2}{5} \times \frac{2}{3}$
2. $2\frac{2}{3} \times \frac{1}{2} \times 1\frac{1}{2}$
3. $1\frac{3}{5} \times \frac{2}{7} \times 1\frac{2}{3}$
4. $\frac{3}{4} \times 2 \times 1\frac{1}{3}$
5. $1\frac{2}{7} \times 4\frac{1}{4} \times 3\frac{1}{2}$

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Find the product for each of the following. Use cancelling as much as possible.

   a. $\frac{1}{2} \times \frac{2}{3}$

   b. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

   c. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$

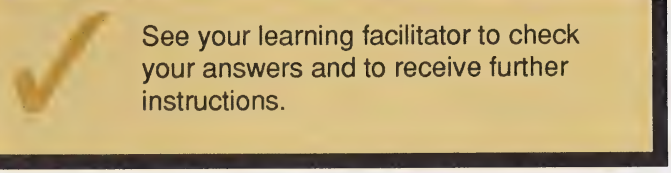
2. What pattern do you notice in Question 1?

3. Use the pattern you discovered to find each of the following products.

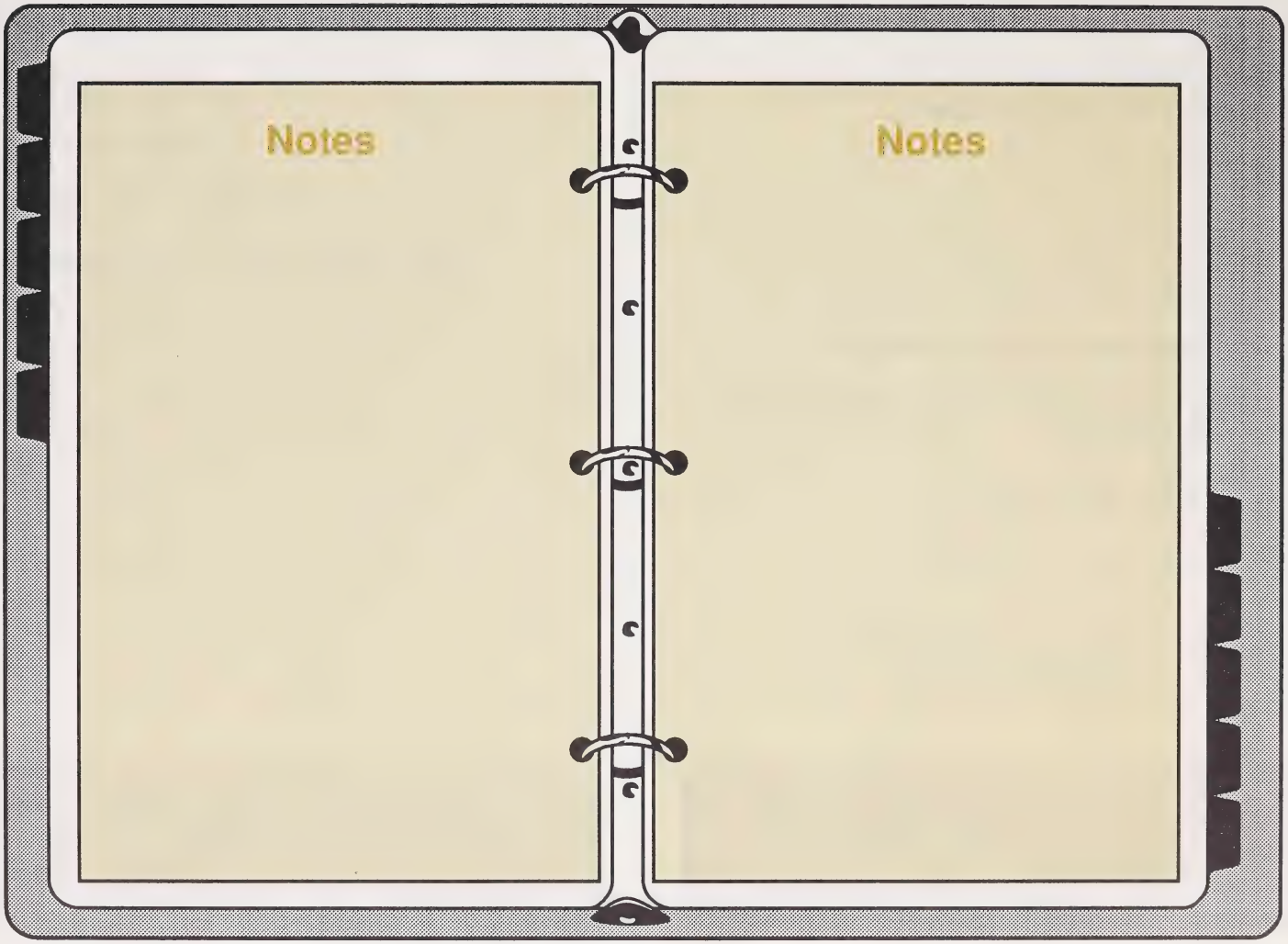
   a. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7}$

   b. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \ldots \times \frac{49}{50}$

This means all the fractions with this pattern between $\frac{4}{5}$ and $\frac{49}{50}$.



See your learning facilitator to check your answers and to receive further instructions.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What Lies Ahead

In this section you will review the rules for the order of operations with whole numbers and extend these rules to include fractions and mixed numbers.

Working Together

Earlier in your study of mathematics, you learned that a series of different operations must be performed in a certain order.

You may have learned the acronym BEDMAS to help you remember the order. The meaning of this acronym is as follows.

B → brackets

E → exponents

D/M → division and multiplication

A/S → addition and subtraction from left to right

In the Introductory Activities you will review the order of operations with whole numbers.

Introductory Activities

Simplify each of the following using the rules for the order of operations.

1. $(5 + 3) \times 8$

2. $(60 - 20) \div 4$

3. $2 \times 8 \div 4 + 20$

4. $3^2 \times 3 + 3$

5. $(14 - 10) + 4^2 \div 8$

6. $\dfrac{2^3 + 5 + 3}{9 \times 2 - 8 - 2}$

7. $2 + 12 \div 2 - 3 \times 2$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Order of Operations for Fractions

The rules for the order of operations as they apply to whole numbers also apply to fractions and mixed numbers.

Example 1: Simplify $\frac{2}{3} + \frac{3}{4} \times \frac{4}{9} + \frac{1}{2}$.

Solution

Estimate the value by rounding each fraction to the nearest $\frac{1}{2}$.

$$\begin{aligned} & \frac{2}{3} + \frac{3}{4} \times \frac{4}{9} + \frac{1}{2} \\ \doteq\ & \frac{1}{2} + 1 \times \frac{1}{2} + \frac{1}{2} \\ \doteq\ & \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \\ \doteq\ & 1\frac{1}{2} \end{aligned}$$

Calculate the exact value.

$$\begin{aligned} & \frac{2}{3} + \frac{3}{4} \times \frac{4}{9} + \frac{1}{2} \\ =\ & \frac{2}{3} + \frac{\cancel{3} \times \cancel{4}}{\cancel{4} \times \cancel{9}} + \frac{1}{2} \\ =\ & \frac{2}{3} + \frac{1}{3} + \frac{1}{2} \\ =\ & 1 + \frac{1}{2} \\ =\ & 1\frac{1}{2} \end{aligned}$$

Example 2: Simplify $\frac{1}{11}\left(\frac{2}{3} + \frac{1}{4}\right) + \frac{1}{2}$.

Solution

Estimate the value by rounding each fraction to the nearest $\frac{1}{2}$.

$$\begin{aligned} & \frac{1}{11}\left(\frac{2}{3} + \frac{1}{4}\right) + \frac{1}{2} \\ \doteq\ & \frac{1}{11}\left(\frac{1}{2} + \frac{1}{2}\right) + \frac{1}{2} \\ \doteq\ & \frac{1}{11} \times 1 + \frac{1}{2} \\ \doteq\ & \frac{1}{11} + \frac{1}{2} \\ \doteq\ & \frac{1}{2} \end{aligned}$$

Calculate the exact value.

$$\begin{aligned} & \frac{1}{11}\left(\frac{2}{3} + \frac{1}{4}\right) + \frac{1}{2} \\ =\ & \frac{1}{11}\left(\frac{8}{12} + \frac{3}{12}\right) + \frac{1}{2} \\ =\ & \frac{1}{11} \times \frac{11}{12} + \frac{1}{2} \\ =\ & \frac{1 \times \cancel{11}}{\cancel{11} \times 12} + \frac{1}{2} \\ =\ & \frac{1}{12} + \frac{1}{2} \\ =\ & \frac{1}{12} + \frac{6}{12} \\ =\ & \frac{7}{12} \end{aligned}$$

Example 3: Simplify $\left(2\frac{1}{2} + 3\frac{1}{3}\right) \div 1\frac{2}{3}$.

Solution

Estimate the value.

$$\left(2\tfrac{1}{2} + 3\tfrac{1}{3}\right) \div 1\tfrac{2}{3}$$
$$\doteq \left(3 + 3\right) \div 2$$
$$\doteq 6 \div 2$$
$$\doteq 3$$

Calculate the exact value.

$$\left(2\tfrac{1}{2} + 3\tfrac{1}{3}\right) \div 1\tfrac{2}{3}$$
$$= \left(\tfrac{5}{2} + \tfrac{10}{3}\right) \div \tfrac{5}{3}$$
$$= \left(\tfrac{15}{6} + \tfrac{20}{6}\right) \div \tfrac{5}{3}$$
$$= \tfrac{35}{6} \div \tfrac{5}{3}$$
$$= \tfrac{35}{6} \times \tfrac{3}{5}$$
$$= \frac{\overset{7}{\cancel{35}} \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \times \underset{1}{\cancel{5}}}$$
$$= \tfrac{7}{2}$$
$$= 3\tfrac{1}{2}$$

Example 4: Simplify $5\frac{5}{6} \div 3\frac{1}{3} - 1\frac{3}{4}$.

Solution

Estimate the value.

$$5\tfrac{5}{6} \div 3\tfrac{1}{3} - 1\tfrac{3}{4}$$
$$\doteq 6 \div 3 - 2$$
$$\doteq 2 - 2$$
$$\doteq 0$$

Calculate the exact value.

$$5\tfrac{5}{6} \div 3\tfrac{1}{3} - 1\tfrac{3}{4}$$
$$= \tfrac{35}{6} \div \tfrac{10}{3} - \tfrac{7}{4}$$
$$= \tfrac{35}{6} \times \tfrac{3}{10} - \tfrac{7}{4}$$
$$= \frac{\overset{7}{\cancel{35}} \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \times \underset{2}{\cancel{10}}} - \tfrac{7}{4}$$
$$= \tfrac{7}{4} - \tfrac{7}{4}$$
$$= 0$$

Practice Exercises

Space for Your Work

1. Simplify each of the following using the rules for the order of operations.

   a. $\frac{1}{4} \times \frac{2}{3} + \frac{9}{10}$

   b. $\frac{2}{5} \div \frac{4}{5} + \frac{1}{3} - \frac{1}{6}$

   c. $\left(\frac{1}{2} + \frac{3}{4}\right) \times \frac{2}{5} \div \frac{3}{8}$

   d. $\frac{3}{4} + \frac{1}{3} \times \left(\frac{2}{3} \div \frac{4}{9}\right)$

   e. $\frac{7}{10} \div \frac{4}{5} \times \frac{17}{20} - \left(\frac{7}{8} + \frac{3}{4}\right)$

2. Simplify each of the following using the order of operation rules.

Space for Your Work

a. $2\frac{1}{2} \div \left(1\frac{3}{4} + \frac{1}{2} \div \frac{1}{3}\right) - \frac{3}{7}$

b. $1\frac{4}{5} \div \left(3\frac{1}{2} + 1\frac{1}{4}\right) - \frac{7}{19}$

c. $1\frac{1}{2} \div 1\frac{1}{3} - 1\frac{1}{3} \div 1\frac{1}{2}$

d. $3\frac{3}{5} \div 2\frac{1}{5} \div \left(5\frac{9}{11} - 4\frac{2}{11}\right)$

e. $\left(1\frac{1}{2} + 2\frac{3}{4}\right) \times \left(2\frac{1}{2} \times \frac{3}{10} \div \frac{1}{4} + \frac{1}{2}\right) \div 2\frac{3}{5} + 3\frac{3}{20}$

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

1. In each of the following, circle the operation to be done first. Do not calculate the answers.

   a. $\frac{1}{2} \times \frac{2}{3} - \frac{4}{5}$

   b. $\left(\frac{3}{4} + \frac{7}{8}\right) - \frac{4}{9}$

   c. $\frac{3}{2} + \frac{1}{4} - \frac{5}{8} \div \frac{1}{10}$

2. In each of the following, number the operations in the order in which they must be done. Do not calculate the answers. Part a. has been done as an example.

   a. $\frac{1}{4} \times \frac{8}{10} - \left(\frac{2}{3} + \frac{3}{5}\right)$

   b. $\frac{1}{6} + \frac{3}{4} \div \frac{1}{2}$

   c. $\frac{4}{3} \times \left(\frac{2}{9} \div \frac{11}{10}\right) - 3\frac{1}{4}$

   d. $\frac{5}{8} \div \frac{3}{2} - \frac{4}{5} + \frac{1}{2} \times \frac{4}{3}$

2. a. (2) (3) (1) $\frac{1}{4} \times \frac{8}{10} - \left(\frac{2}{3} + \frac{3}{5}\right)$

3. Is the answer for each of the following correct? Answer by writing **yes** or **no**. Where the choice is *no*, correct the answer for that question.

Space for Your Work

a. $\frac{1}{2} \div \frac{5}{4} + \frac{2}{3} = 1\frac{1}{15}$

b. $\frac{7}{8} + \frac{3}{4} \div \left(\frac{1}{2} \times \frac{3}{4}\right) = 4\frac{1}{3}$

c. $\frac{3}{2} \times \frac{4}{5} - \frac{5}{6} \div \frac{5}{3} + \frac{1}{4} = \frac{9}{20}$

d. $\left(\frac{1}{3} + \frac{1}{2}\right) \div \frac{7}{6} \times 3 = 2\frac{1}{7}$

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Solve these problems.

1. The body of a dinosaur is 32 m in length. The head of the dinosaur is $\frac{1}{8}$ as long as its body. The tail is as long as the head plus $\frac{1}{4}$ of the body. How long is the dinosaur altogether?

2. Pam had a bag of candies. She gave $\frac{1}{2}$ of the candies to her sister and ate one herself. After that she gave away $\frac{1}{2}$ of what she had left and ate one herself. Finally she gave away $\frac{1}{2}$ of what she had left to her friend and ate one herself. If she had two candies left and if all the candies were whole, how many candies did she have originally?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn to interpret rational numbers.

In this section you will learn these terms.

- rational numbers
- irrational numbers

Working Together

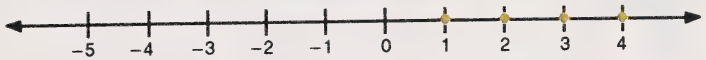
Our number system is made up of several types of numbers.

Types of Numbers	Symbol
Natural numbers	N
Whole numbers	W
Integers	I
Rational numbers	Q

Note

The symbol for the rational numbers is Q, not R. This is because R is used for a still larger group of numbers called the real numbers. You will learn more about the real numbers in Grade 10 math.

The Natural Numbers



The natural numbers include the counting numbers.

1, 2, 3, 4, 5, 6, ...

There is a least number, which is 1, but there is no greatest number in this group.

There is no natural number between 1 and 2, 2 and 3, 3 and 4, and so on.

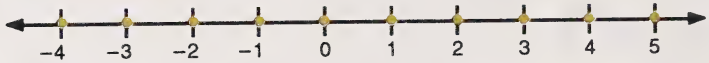
The Whole Numbers

The whole numbers include all the natural numbers plus 0.

There is a least number, but there is no greatest number in this group.

There is no whole number between 0 and 1, 1 and 2, 2 and 3 and so on.

The Integers



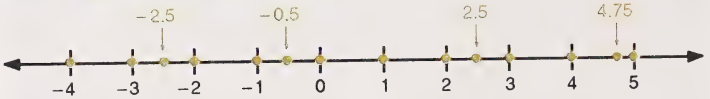
The integers include all the whole numbers plus their opposites.

..., – 3, – 2, – 1, 0, 1, 2, 3, ...

There is no least number and no greatest number.

There is no integer between – 3 and – 2, – 2 and – 1, – 1 and 0, and so on.

The Rational Numbers

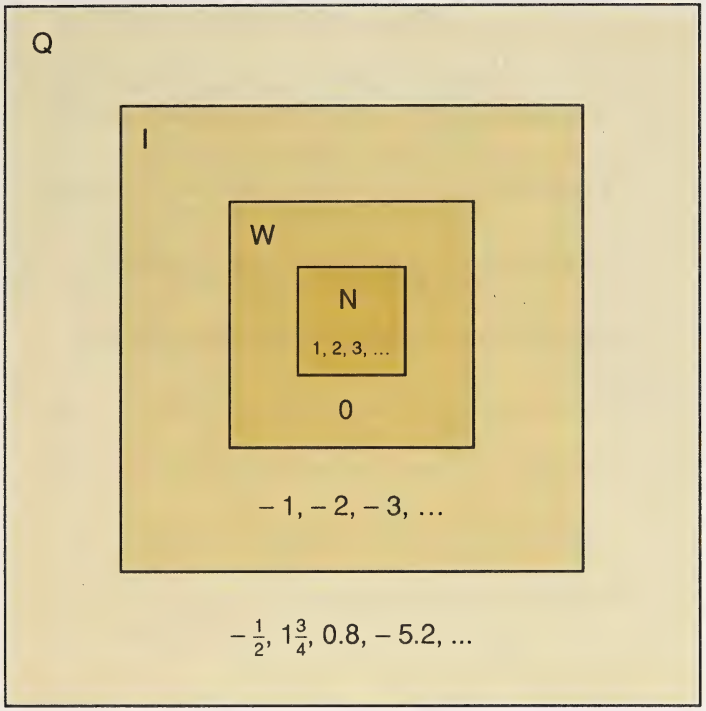


The rational numbers include the integers, the positive and negative fractions, and the decimal numbers.

There is no least number and no greatest number.

The rational numbers are dense. This means that there is an infinite number of rational numbers between any two rational numbers.

This diagram shows how the different groups of numbers are related.



- 1 is a natural number, a whole number, an integer, and a rational number.
- 0 is a whole number, an integer, and a rational number. 0 is not a natural number.
- – 1 is an integer and a rational number, but not a whole number or a natural number.
- $-\frac{1}{2}$ is a rational number, but not an integer, whole number, or a natural number.
- $1\frac{3}{4}$ is a rational number, but not an integer, whole number, or a natural number.
- 0.8 is a rational number, but not an integer, whole number, or a natural number.
- – 5.2 is a rational number, but not an integer, whole number, or a natural number.

The rational numbers, which are not also integers, whole numbers or natural numbers, are either positive and negative fractions, or positive and negative decimal numbers.

Test for Rational Numbers

All rational numbers can be expressed as a quotient of two integers.

Each of the following is a rational number since it can be expressed as a quotient of two integers.

- 4 is a rational number since it can be written as $\frac{+4}{+1}$.
- 0 is a rational number since it can be written as $\frac{0}{+1}$.
- -2 is a rational number since it can be written as $\frac{-2}{+1}$.
- 0.5 is a rational number since it can be written as $\frac{+5}{+10}$.
- -0.8 is a rational number since it can be written as $\frac{-8}{+10}$.
- $\frac{1}{2}$ is a rational number since it can be written as $\frac{+1}{+2}$.
- $-\frac{3}{4}$ is a rational number since it can be written as $\frac{-3}{+4}$.
- $1\frac{1}{2}$ is a rational number since it can be written as $\frac{+3}{+2}$.
- $-1\frac{1}{2}$ is a rational number since it can be written as $\frac{-3}{+2}$.

Introductory Activities

Space for Your Work

1. Answer each of the following by writing **true** or **false**.

   a. Zero is not a rational number.

   b. All integers are rational numbers.

   c. All rational numbers are integers.

   d. All whole numbers are integers.

   e. All integers are whole numbers.

   f. All negative numbers are rational numbers.

   g. All rational numbers are negative numbers.

   h. All whole numbers are rational numbers.

   i. All rational numbers are whole numbers.

   j. All positive and negative proper fractions are rational numbers.

2. Complete this chart by identifying each number as a natural number, whole number, integer, or rational number. Some of the given numbers fit into more than one category.

   a. 1

   b. $\frac{3}{5}$

   c. -0.125

   d. -3

   e. 0

Space for Your Work

N	W	I	Q

3. Express each of the following as a quotient of two integers.

Space for Your Work

a. 5

b. -8

c. 0.44

d. -0.06

e. $\frac{3}{5}$

f. $-\frac{2}{7}$

g. $1\frac{1}{3}$

h. $-1\frac{3}{4}$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Comparing Rational Numbers

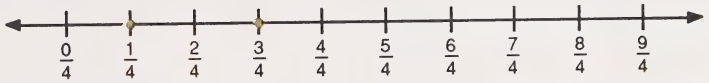
There are several ways to decide whether one rational number is less than, equal to, or greater than another rational number.

Method 1

One way is to picture them on a number line. Any number located to the left of a given number is less than that number. Any number located to the right of a given number is greater than that number.

Example 1: Compare $\frac{1}{4}$ and $\frac{3}{4}$ and tell which one is greater in value.

Solution

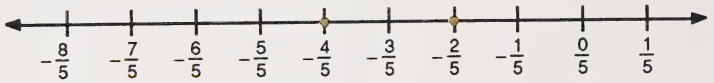


$\frac{3}{4}$ is to the right of $\frac{1}{4}$.

So, $\frac{3}{4} > \frac{1}{4}$.

Example 2: Compare $-\frac{2}{5}$ and $-\frac{4}{5}$ and tell which one is greater in value.

Solution

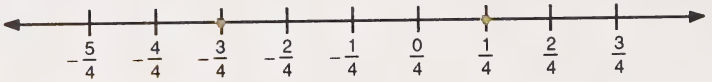


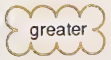
$-\frac{2}{5}$ is to the right of $-\frac{4}{5}$.

So, $-\frac{2}{5} > -\frac{4}{5}$.

Example 3: Compare $-\frac{3}{4}$ and $-\frac{1}{4}$ and tell which one is greater in value.

Solution



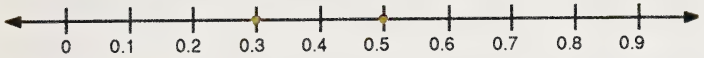


$\frac{1}{4}$ is to the right of $-\frac{3}{4}$.

So, $-\frac{1}{4} > -\frac{3}{4}$.

Example 4: Compare 0.3 and 0.5 and tell which one is greater in value.

Solution



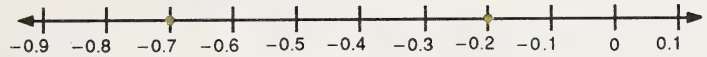
lesser greater

0.5 is to the right of 0.3.

So, 0.5 > 0.3.

Example 5: Compare – 0.2 and – 0.7 and tell which one is greater in value.

Solution



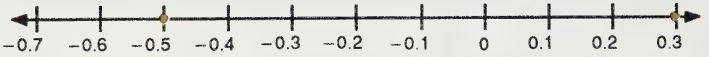
lesser greater

– 0.2 is to the right of – 0.7.

So, – 0.2 > – 0.7.

Example 6: Compare 0.3 and – 0.5 and tell which one is greater in value.

Solution



lesser greater

0.3 is to the right of – 0.5.

So, 0.3 > – 0.5.

Method 2

Here is another method of comparing rational numbers. Use the fraction form. If the denominators are the same, compare the numerators. If the denominators are not the same, make changes so that they are, and then compare the numerators.

Example 1: Which is greater, $\frac{2}{5}$ or $\frac{3}{5}$?

Solution

The denominators are the same.

Since $3 > 2$, then $\frac{3}{5} > \frac{2}{5}$.

Example 2: Which is greater, $\frac{2}{5}$ or $\frac{3}{7}$?

Solution

The denominators are different. Use equivalent rational numbers which have the same denominator.

$\frac{2}{5} = \frac{14}{35}$ (×7 on both)

$\frac{3}{7} = \frac{15}{35}$ (×5 on both)

Since $15 > 14$, then $\frac{3}{7} > \frac{2}{5}$.

Be careful when comparing negative numbers.

Example 3: Which is greater, $-\frac{3}{8}$ or $-\frac{5}{8}$?

Solution

The denominators are the same.

Since $-3 > -5$, then $-\frac{3}{8} > -\frac{5}{8}$.

Example 4: Which is greater, $-\frac{5}{7}$ or $-1\frac{1}{3}$?

Solution

The denominators are different. Use equivalent rational numbers which have the same denominator.

$1\frac{1}{3} = -\frac{4}{3}$

$-\frac{5}{7} = -\frac{15}{21}$ (×3 on both)

$-\frac{4}{3} = -\frac{28}{21}$ (×7 on both)

Since $-15 > -28$, then $-\frac{5}{7} > -1\frac{1}{3}$.

Method 3

Still another method is to use the decimal number forms to make the comparisons.

Example 1: Which is larger, $\frac{3}{4}$ or 0.8?

Solution

$\frac{3}{4} = 0.75$ $\qquad$ $0.8 = 0.80$

Compare the digits in each place. Work from left to right.

0.75
↑ ↑
same different
↓ ↓
0.80

Since $8 > 7$, and $0.8 > 0.75$,

$0.8 > \frac{3}{4}$.

Be careful when comparing negative numbers.

Example 2: Which is larger, -1.3 or $-1\frac{1}{4}$?

Solution

$-1.3 = -1.30$ $\qquad$ $-1\frac{1}{4} = -1.25$

Compare the digits in each place. Work from left to right.

−1.30
↑ ↑
same different
↓ ↓
−1.25

Since $-2 > -3$, and $-1.25 > -1.3$,

$-1\frac{1}{4} > -1.3$.

Practice Activities

Space for Your Work

Using each pair of rational numbers, decide which of the two is larger.

1. $\frac{3}{7}$ and $\frac{6}{7}$
2. $\frac{5}{8}$ and $\frac{3}{4}$
3. $1\frac{7}{8}$ and $1\frac{9}{10}$
4. $-\frac{3}{5}$ and $-\frac{4}{5}$
5. $-2\frac{3}{10}$ and $-2\frac{1}{10}$
6. 0.75 and 0.3
7. -1.03 and -1.12
8. -2.75 and $-2\frac{3}{5}$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Earlier in this module you learned about terminating and repeating decimal numbers.

All decimal numbers that terminate can be expressed as a quotient of two integers.

Examples

- $0.7 = \frac{7}{10}$
- $-0.3 = -\frac{3}{10}$
- $3.125 = 3\frac{125}{1000} = 3\frac{1}{8} = \frac{25}{8}$
- $-5.625 = -5\frac{625}{1000} = -5\frac{5}{8} = -\frac{45}{8}$

So, all decimal numbers that terminate are rational numbers.

All decimal numbers that have a repeating digit or a repeating block of digits can be expressed as a quotient of two integers.

Examples

- $0.\dot{6}\dot{3} = \frac{7}{11}$
- $-0.1\dot{6} = -\frac{1}{6}$
- $4.\dot{3} = 4\frac{1}{3} = \frac{13}{3}$
- $-1.\dot{2} = -1\frac{2}{9} = -\frac{11}{9}$

So, all decimal numbers that have a repeating digit or block of repeating digits are rational numbers.

Some decimal numbers neither repeat nor terminate. These decimal numbers cannot be expressed as a quotient of two integers. These decimal numbers are not rational numbers.

Examples

- $-0.101\ 001\ 001\ \ldots$
- $3.141\ 592\ 653\ \ldots$

Decimal numbers that neither repeat nor terminate are said to be **irrational numbers**.

Concluding Activities

Is each of these decimal numbers a rational number? Explain why or why not.

1. 0.8
2. 0.924
3. $0.2\dot{3}$
4. $-0.\dot{5}$
5. – 0.75
6. 2.123 122 312 222 …

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In Module 3 you have learned these skills.

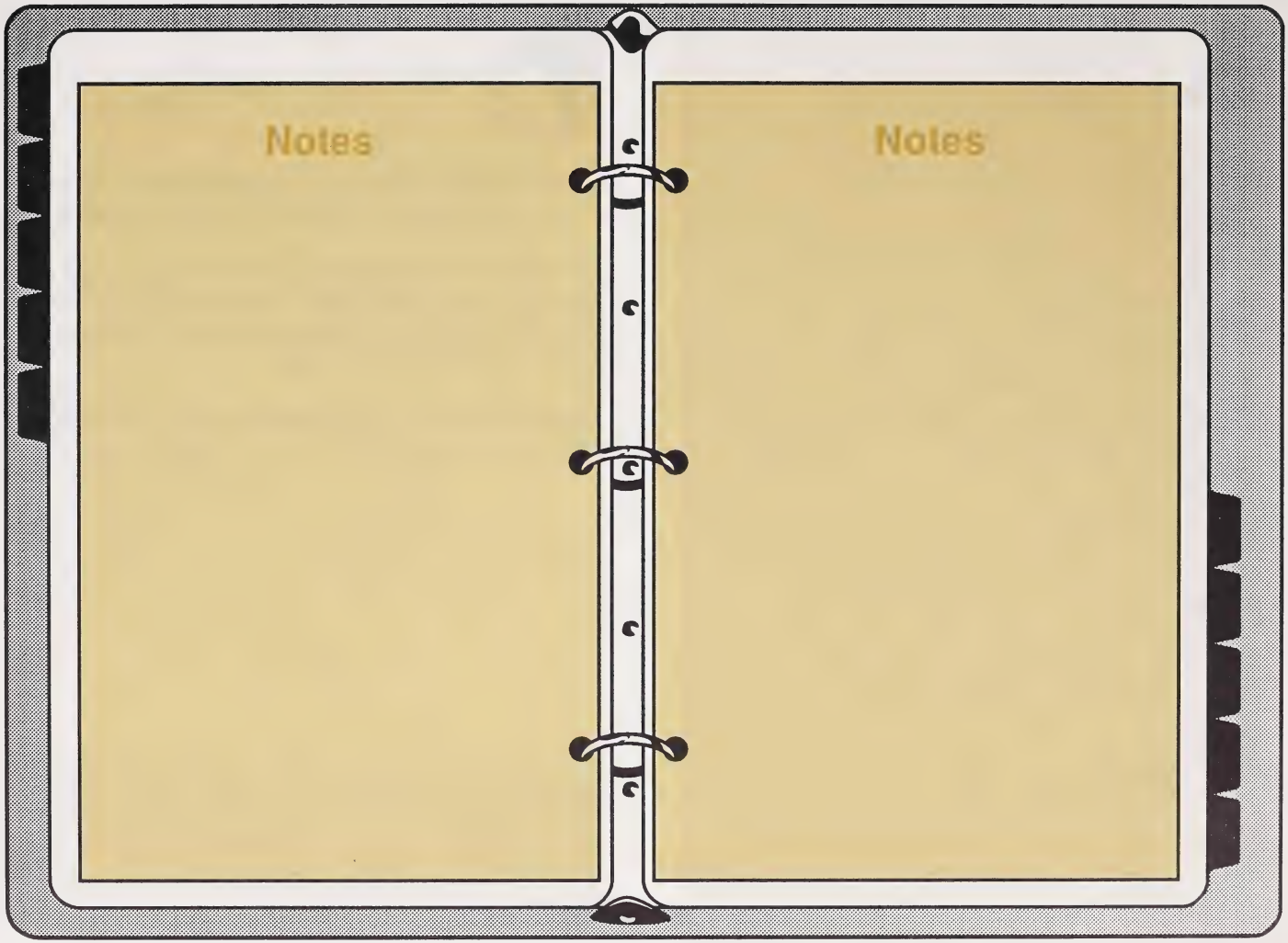
- interpreting a fraction
- writing equivalent fractions
- writing basic fractions
- writing mixed numbers in fraction form
- writing fractions as mixed numbers
- expressing a fraction as a decimal number
- expressing terminating decimal numbers as fractions
- comparing fractions
- estimating sums, differences, products, and quotients of fractions and mixed numbers
- calculating sums, differences, products, and quotients of fractions and mixed numbers using paper and pencil methods and using calculators
- adding, subtracting, multiplying, and dividing fractions and mixed numbers mentally
- performing a series of different operations on fractions and mixed numbers
- interpreting rational numbers

Working Together

Sections 3 to 17 dealt with fractions. Section 18 dealt with number systems. Review the skills you have learned.

Turn to Section 2 and review the pretest. Correct any errors you may have made at the time you did the pretest. You may be pleasantly surprised to discover how much you have learned!

This review will help you prepare for the assignment in the module conclusion.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MODULE CONCLUSION

What Lies Ahead

In this section you will complete the module assignment.

Working Together

Now that you have completed the Module Booklet, you are ready for the Module Assignment.

Module Assignment

Turn to the Assignment Booklet and complete the Module Assignment independently. You may use reference materials but do not get help from anyone.

APPENDIX

GLOSSARY

Ascending order: numbers are arranged in order of size from lowest to highest

Basic fraction: the simplest form of a fraction

Borrowing: regrouping to allow subtraction

$$1\frac{1}{4} = \frac{5}{4}$$
$$-\frac{3}{4} = -\frac{3}{4}$$
$$\frac{2}{4} = \frac{1}{2}$$

Cancelling: the division of the numerators and the denominators by the same number

$$\frac{3}{4} \times \frac{2}{9} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times \underset{3}{\cancel{9}}}$$
$$= \frac{1}{6}$$

Carrying: regrouping in addition

$$1\frac{3}{4}$$
$$+\,2\frac{3}{4}$$
$$3\frac{6}{4} = 4\frac{2}{4} = 4\frac{1}{2}$$

Common fraction: a fraction in which the numerator and denominator are whole numbers

Complex fraction: a fraction in which the numerator or the denominator is a fraction

$$\frac{\frac{3}{4}}{2}, \quad \frac{5}{\frac{1}{2}}, \quad \frac{\frac{1}{2}}{\frac{3}{4}}$$

Cross products: When comparing fractions, cross products can be used as a shortcut for finding equivalent fractions and comparing numerators.

Compare $\frac{2}{5}$ and $\frac{3}{7}$.

Equivalent Fractions	Cross Product
$\frac{2}{5} = \frac{14}{35}$ ($\times 7$) and $\frac{3}{7} = \frac{15}{35}$ ($\times 5$)	$7 \times 2 = 14$ and $5 \times 3 = 15$
$14 < 15$	$14 < 15$
So, $\frac{2}{5} < \frac{3}{7}$.	So, $\frac{2}{5} < \frac{3}{7}$.

Denominator: the bottom number of a fraction

$\frac{2}{3}$ ← denominator

Descending order: numbers are arranged in order of size from greatest to least

Equivalent fractions: fractions that represent the same number

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$ and so on.

Identity property: The sum of any number and 0 is that number. The product of any number and 1 is that number.

$0 + 4 = 4$

$1 \times 5 = 5$

Improper fraction: a fraction in which the numerator is larger than or equal to the denominator

$\frac{3}{2}, \frac{4}{4}, \frac{5}{3}, \frac{2}{2}, \frac{7}{4}$, and $\frac{9}{5}$ are a few improper fractions.

Integer: any number in the group consisting of the natural numbers, their opposites, and 0

$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Irrational numbers: decimal numbers which do not terminate and do not repeat

0.123 456 789 101 112 ...

0.122 111 2222 ...

Mixed number: a number greater than 1 which is expressed as a whole number and a fraction

$1\frac{1}{2}, 2\frac{3}{4}, 5\frac{1}{10}$ are mixed numbers.

Natural number: any number in the group of counting numbers

1, 2, 3, 4, 5, ...

Numerator: the top number of a fraction

$\frac{2}{3}$ ← numerator

Order of operations: the rules which are followed in simplifying a mathematical expression with several operations

Place value: the value given to the place that a digit appears in a numeral

In 0.205, the 5 is in the thousandths place.

Proper fraction: a number less than 1 which is expressed as a fraction

$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$ are proper fractions.

Rational number: a number that can be expressed as a quotient of two integers

Repeating decimal number: a repeating decimal number has a digit or blocks of digits which repeat

$0.333\ldots$, or $0.\dot{3}$, or $0.\dot{3}$ is a repeating decimal.

$0.125\ 125\ldots$, or $0.\dot{1}2\dot{5}$, or $0.\overline{125}$ is a repeating decimal.

Reciprocal: The reciprocal of a given number is the number whose product with the given number is 1.

The reciprocal of 3 is $\frac{1}{3}$.

The reciprocal of $\frac{7}{8}$ is $\frac{8}{7}$ or $1\frac{1}{7}$.

Simplest form: A fraction is expressed in simplest form if the numerator and the denominator have no common factor other than 1.

$\frac{2}{3}$, $\frac{3}{10}$ and $\frac{5}{6}$ are in simplest form.

Terminating decimal number: a decimal number that has a finite number of digits

0.3 and 0.25 are terminating decimals.

Vinculum: the bar that separates the numerator and the denominator in a fraction

$\frac{3}{4}$ ← vinculum

Whole number: any number in the group consisting of the natural numbers and 0

0, 1, 2, 3, 4, ... are all whole numbers.

MATH 8

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